A NEW STUDY TO PREDICT THE PERMEABILITY OF CONCRETE MATERIALS

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Summary: *The aim of this article is to develop a new analytical and numerical study to determine the permeability of porous material (as concrete, soil mass, aggregates ...). The materials in this situation are periodic and defined by an elementary cell. The numerical results for the permeability are obtained within the framework of the periodic homogenization technique established in the literature, using the open source finite element code.*

Keywords: *permeability; concrete structure; durability; finite element.*

I. INTRODUCTION

Durability is the ability of a structure to withstand various forms of attack from the environment. For concrete structures, the durability is largely a function of the aggressive agents to penetrate into the porous network of concrete. Two physical quantities used to characterize the ability of concrete to resist the intrusion of aggressive agents: permeability and diffusion. Both magnitudes correspond to two distinct mechanisms of material transport: permeability describes a flow that occurs under pressure gradient and diffusion transport at the molecular level in concentration gradient. There are therefore macroscopic "effective" material properties of porous media having an open interconnected porosity. It's really to show that pore structure of concrete plays a significant role in physical and chemical deteriorations of concrete. In context of the research topic of durability of concrete structure has developed at UTC, we are interested to study the relationship between permeability and porous structure of concrete. To do this, we divide this subject into two parts: the first concerning how to predict the permeability of a material from descriptions of its porous structure and second concerning the applications in concrete structures.

In this paper, we focus on this first subject. In fact, this area is the subject of much research. Some analytical studies are based on the fact that the configuration is simple and those results are not really exact but they're good enough to evaluate the influence of some aspects [1]. Some other models that interested in solving the equations Navier-Stokes by numerical tools are quite complicated and expensive [2].

Therefore, we are interested in develop a new approach to simulate the relationship between permeability and porous structure. It necessary that this new approach is simple but it's also powerful to adapt the demands from engineers. That is the subject of the present paper.

II. GOVERNING EQUATIONS

2.1. Darcy's law

Darcy was the first to address the question that describes the flow of a fluid through the pore space of a porous medium. He studied the water flow through a cylindrical sample of sand by controlling the head difference between the upper and lower faces of the cylinder. While keeping the difference in water levels constant during the experiment, Darcy measured the flow D across the section S of the sample, and obtained the remarkable result that Q = D/S is proportional to the ratio $\Delta H/L$ of the head difference over the height L of the sample:

$$Q = \frac{D}{S} = K\frac{\Delta H}{L} \tag{1}$$

The coefficient K which appears in (1) represents the permeability of the sample in the direction of the flow. Generally, Darcy's observations suggest that the fluid flow is proportional to the pressure gradient and oriented in the opposite direction, which can be captured by the following macroscopic vector formulation of the transport law for porous media (porosity φ):

$$\underline{Q} = K(-\underline{\nabla}P + \rho\underline{g}) \tag{2}$$

Where $\underline{Q}$ is referred to as the filtration velocity, P is the (macroscopic) fluid pressure, $\rho\underline{g}$ is the fluid unit weight ($\underline{g}$ the nominal gravitational acceleration). The permeability coefficient K is not an intrinsic property of the porous medium, but depends also on the fluid. An intrinsic permeability, i.e. one that depends only on the pore geometry and not on the saturating fluid phase, can be defined in the following form:

$$K = \frac{K'}{\mu} \tag{3}$$

Where μ is shear viscosity of an incompressible Newtonian fluid and dimension of $[K']$ = Length^2.

In the next section, we will use the micro-to-macro approach based on the periodic homogenization technique and translates the Stokes equations that describe the equilibrium and constitutive behavior of the fluid at the micro scale into the macroscopic Darcy's law to derive the macroscopic permeability as a function of the microscopic pore morphology and the fluid viscosity. The main ideas developed here are inspired by the work of Dormieux et al [3].

2.2. Stokes equations in framework of periodic homogenization

We consider here a porous medium is made up of a rigid solid in domain Ω^s and saturated by a homogeneous incompressible Newtonian fluid defined by a viscosity coefficient μ in domain Ω^f. For simplicity, we will assume that the Reynolds number is small enough so that inertia effects can be neglected in the momentum balance equation of the fluid. We also disregard the gravity forces and the solid phase is rigid. The microscopic flow is therefore subjected to the following set of Stokes equations:

$$\begin{cases} -\nabla_z p + \mu \Delta \underline{v} = 0 & : (\Omega^f) \\ \mathrm{div}\underline{v} = 0 & : (\Omega^f) \\ \underline{v} = 0 & : (\partial\Omega^s = \Omega^f \cap \Omega^s) \end{cases} \qquad (4)$$

The microscopic stress state in the fluid is related to the strain rate tensor $\underline{\underline{d}}$, defined as the symmetric part of the microscopic velocity gradient $\underline{v}$, by the classical state equation of fluid mechanics:

$$\underline{\underline{\sigma}} = -p(\underline{z})\underline{\underline{1}} + 2\mu\underline{\underline{d}}(\underline{z}) \qquad \text{with} \qquad \underline{\underline{d}}(\underline{z}) = \frac{1}{2}(\nabla\underline{v} + {}^t\nabla\underline{v}). \qquad (5)$$

Then, we consider Representative Volume Element of porous media (Ω) imposed by a macroscopic pressure gradient $\underline{\nabla}P$. The effective permeability is determined as the proportional coefficient relating the macroscopic filtration velocity $\underline{Q}$ and the negative of macroscopic pressure gradient $\underline{\nabla}P$ (consult (2)). It is necessary to recall that the macroscopic filtration velocity is equal to the volume average of microscopic velocity over Ω:

$$\underline{Q} = \frac{1}{|\Omega|}\int_{\Omega} \underline{v}(\underline{z})dz = \langle \underline{v}(\underline{z}) \rangle_{\Omega} \qquad (6)$$

And a similar relationship:

$$P = \frac{1}{|\Omega|}\int_{\Omega} p(\underline{z})dz = \langle p(\underline{z}) \rangle_{\Omega} \qquad (7)$$

Where $\underline{z}$ denotes a material point in Ω.

Furthermore, the geometry of the microstructure is periodic and the elementary cell (U) is assumed to be a parallelepiped one (their lengths are denoted by a_1, a_2, a_3) that comprises all the geometrical information necessary for the determination of the microscopic characteristics. For this elementary cell, we have a periodic field of velocity $\underline{v}(\underline{z})$:

$$\forall n_1, n_1, n_1 \in N \qquad \underline{v}(\underline{z} + n_1 a_1 \underline{e}_1 + n_2 a_2 \underline{e}_2 + n_3 a_3 \underline{e}_3) = \underline{v}(\underline{z}) \qquad (8)$$

Where $(\underline{e}_1, \underline{e}_2, \underline{e}_3)$ denotes an orthonormal frame. From [4], we look for an extension of the pressure p in the whole cell (solid and fluid domains) in the form:

$$p(\underline{z}) = \underline{\nabla}P.\underline{z} + f(\underline{z}) \qquad (9)$$

Where $f(\underline{z})$ is a periodic function.

The periodicity condition of the microscopic velocity field $\underline{v}(\underline{z})$ implies that the average of $\underline{v}(\underline{z})$ over the domain U is the same in all cells contained in the considered Representative Volume Element

$$\underline{Q} = < \underline{v}(\underline{z}) >_{W} = < \underline{v}(\underline{z}) >_{U} \qquad (10)$$

This indicates that the solution of the problem from (2) to (9) be determined from the solution over the domain U governed by the equations as

$$\begin{cases} \mathrm{div}\underline{\underline{s\%}} - \underline{Ñ}P = 0 & :(U^f) \\ \underline{\underline{s\%}} = -f(\underline{z})\underline{\underline{1}} + 2m\underline{\underline{d}}(\underline{z}) & :(U^f) \\ \underline{\underline{d}}(\underline{z}) = \frac{1}{2}(Ñ\underline{v} + {}^tÑ\underline{v}) & :(U^f) \\ \mathrm{div}\underline{v} = 0 & :(U^f) \\ \underline{v} = 0 & :(¶U^{fs} = U^f \, Ç U^s) \end{cases} \qquad (11)$$

Where the stress field $\underline{\underline{s\%}} = \underline{\underline{s}} + \underline{Ñ}P.\underline{z}\underline{\underline{1}}$ which in contrast to the real stress field $\underline{\underline{s}}$ is periodic.

It is interesting to note the formal analogy of (11) with a problem of incompressible elasticity with the gravity forces $\underline{Ñ}P$ being the loading parameter. Indeed, it suffices to replace the velocity $\underline{v}$ by a displacement field $\underline{x}$, and the strain rate $\underline{\underline{d}}$ by the strain $\underline{\underline{e}}$ associated with $\underline{x}$. Exploring this analogy, we can solve the problem (11) by the well-known methods applying for elastic solid problem developed in the literature [5].

III. NUMERICAL APPLICATIONS

The numerical results for the solution (11) are obtained within the framework of the periodic homogenization technique established in the literature, using the open source finite element code (CASTEM). The steps are as following: Create the respective finite element meshes; Provide the respective material and mechanical properties; Establish the periodic boundary conditions; Impose a gravity forces (value is 1); The permeability is equal to the volume average of microscopic velocity over elementary cell.

For particular calculations, we examine two examples. Example 1 defines the permeability of the two-dimensional porous structure made by square array of circle disks (Figure 1) while example 2 verifies the classical Poiseuille solution (figure 2). Relationship between normalized permeability and porosity are reported respectively in figures 3 and 4.

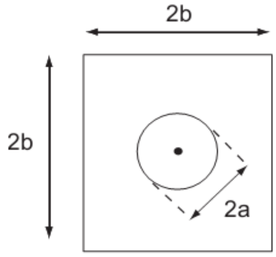

Figure 1. *Flow in a square array of circle disks*

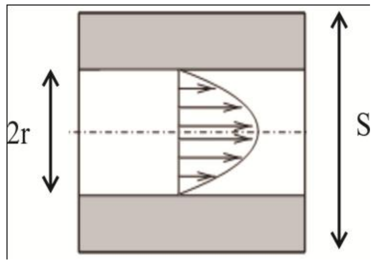

Figure 2. *Flow in a cylindrical pore*

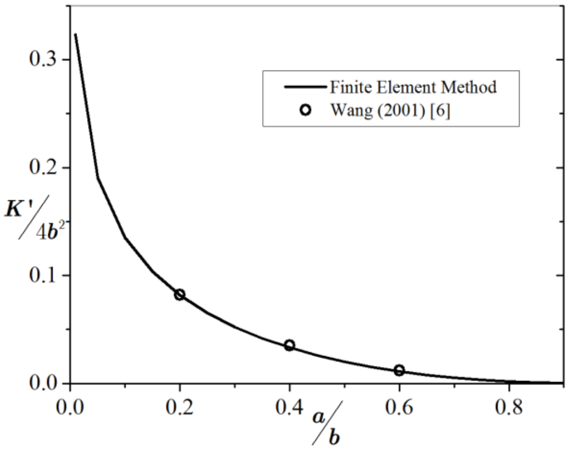

Figure 3. *Relationship between the normalized permeability and porosity in a square array of circle disks*

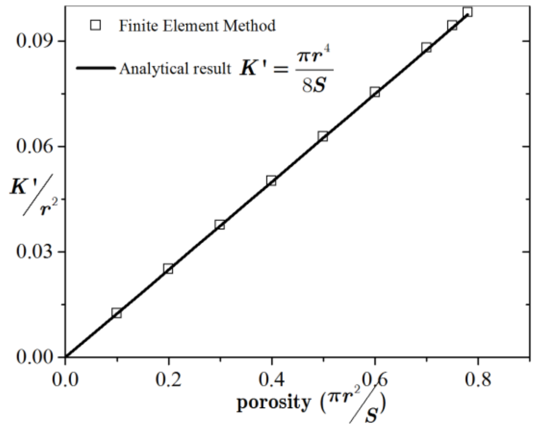

Figure 4: *Relationship between the normalized permeability and porosity in a cylindrical pore space*

IV. CONCLUSION

Finite element simulations for periodic arrays of circle inclusions and Poiseuille flow showed agreeable results. This indicates the efficiency of the approach. Broader development of the approach concerning the influence of concrete structures shall be followed.

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