DEVELOPMENT OF GENERALIZED LIMIT EQUILIBRIUM METHOD FOR THE FAILURE OF RETAINING WALLS UNDER SEISMIC LOADINGS

XUAN BINH LUONG; VIKHONE SAYNHAVONG
THANH THUY HOANG
Department of Civil Engineering
University of Transport and Communications, Vietnam
MEIKETSU ENOKI
Department of Civil Engineering
Tottori University, Japan

Abstract: *The classical theory of plasticity, represented by Kötter's equation, has been established for static problems [1]. Although this theory can be easily extended for dynamic plasticity problems by introducing accelerations as inertia forces, up to now no researcher has done this because of the difficulty in determining the acceleration distribution within the body when the failure occurs. The objective of this research is to develop Generalized Limit Equilibrium Method (GLEM) with the introduction of continuity condition of acceleration to investigate the following cases of potential failures of retaining structures: active failure, foundation-like failure, and slope-like failure, under seismic loadings, where the GLEM is one of the limit equilibrium methods proposed by Enoki at al. The theoretical formulation of the method, the illustrative examples, and the comparisons between the results of the proposed method and other methods are demonstrated.*

Keywords: *Earth pressure, earthquake, limit equilibrium method, slope failure, rigid-plastic*

I. INTRODUCTION OF GLEM

1.1. Outline of Generalized Limit Equilibrium Method for Static Problems

Developing the limit equilibrium method (LEM), the authors have proposed the generalized limit equilibrium method, which can obtain a force field satisfying the equilibrium condition on every soil block and failure condition both on the bottom plane and the inter-block plane of the block (Figure. 1). Consequently, the GLEM can be considered as an approximation method to obtain the necessary condition of SLM. This method has the following features:

Quadrangle or triangle blocks as well as slices can be treated.

Safety factors are defined both on the main sliding surface and on inter-block planes.

Circular sliding surface as well as non-circular sliding surface can be treated.

All types of plasticity problems can be expressed in a single formulation.

The GLEM can be applied to analyze all types of static problems such as slope stability, earth pressure, and bearing capacity. For every static plastic problem, a number of examples of the calculation were taken with the use of GLEM and many other methods. The comparisons showed that the results obtained by GLEM agree with those obtained by theoretical analysis. Another paper by the authors (Enoki et al., 1991) [2] should be referred to for the details of the GLEM.

1.2. Outline of Generalized Limit Equilibrium Method for Dynamic Problems

Newmark [3] proposed, for the first time, a displacement analysis method to evaluate the effects of earthquakes on the stability of slopes. The method was then developed by Chang [4] and other researchers [5]. In Newmark's method, the critical state in which the failure begins to occur is determined by the pseudo-static analysis. When the earthquake-induced acceleration exceeds the critical value, the failed soil mass is considered to slide along the slip surface as a whole rigid body. The residual displacement can be determined by integrating the relative acceleration.

Based on Newmark's concept, the authors have been developing a method to analyze the motion of earth structures. In this method, the computing model of a foundation is shown in figure 1. When the earthquake-induced acceleration reaches a certain critical value, incipient failure occurs and many slip planes appear within the body. This critical acceleration can be obtained by ordinary pseudo-static analysis in which the acceleration of every part of the structure is the same with the input acceleration. When the seismic acceleration exceeds the critical value, the failed soil mass is then considered as a rigid-plastic block system, in which the surrounding surfaces of the block are just the slip planes. The rigid blocks will move relative to each other and to the base ground along the slip planes. Across a slip plane (figure 2) the component of acceleration normal to the slip plane is continuous, the component parallel to the slip plane is discontinuous but the shear stress on the slip plane corresponding to the shear strength is transmitted. This continuity condition of acceleration is combined with "Generalized Limit Equilibrium Method (GLEM)" to analyze the motion of the earth structures in dynamic cases. The residual displacements of every block in both vertical and horizontal directions can be computed by integrating twice the relative acceleration of the block.

The proposed method permits the analysis of all types of dynamic plastic problems such as: bearing capacity and motion of foundations, failure and motion of slopes, earth pressure and motion of retaining walls. Any types of sliding surface can be treated by dynamic GLEM.

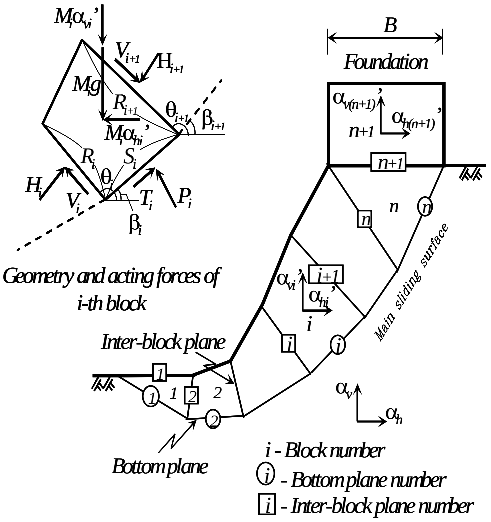


Figure 1. A block system of a foundation in earthquake motion and equilibrium of the i - th block

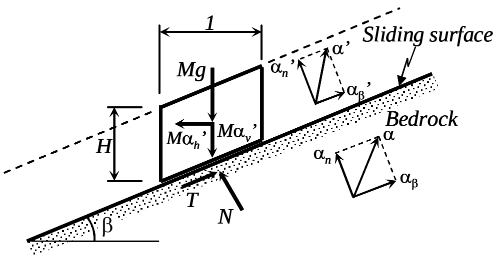


Figure 2. *A sliding block model*

II. FAILURE MECHANISMS OF RETAINING WALLS UNDER SEISMIC LOADINGS

Figures 3, 4 and 5 show the computing models of a retaining wall subjected to seismic loadings corresponding to three cases of failures: active failure (a), failure of retaining wall as a foundation problem (foundation-like failure) (b), and failure of the wall as a slope problem (slope-like failure) (c). In mode (a), the wall is considered to move outwards relative to sub-base, and causes the active earth pressure. In mode (b), the base supporting the wall is failed and both wall and sub-base slide outwards. In mode (c), both the sub-base and backfill are failed and the system slides outwards. The failed soil mass is considered as a rigid-plastic block system. Either triangular or quadrangular blocks can be used.

III. FORMULATION OF DYNAMIC GLEM FOR THE FAILURES OF RETAINING WALLS

Before the sliding occurs, the acceleration of every soil block is the same as the acceleration of the sub-base. The equilibrium equations of every block, the failure conditions on the inter-block planes and bottom planes are used to obtain the force field as presented in the formulation of dynamic GLEM, for the detail the [6] should be refered to.

When the sliding occurs, the accelerations of blocks are different from each other and from the sub-base. The equilibrium equations of every block, the failure conditions, and continuity conditions of acceleration on both inter-block planes and bottom planes are used. The number of unknowns and the number of equations are shown in Table 1. The sliding acceleration of the wall is minimized to obtain the geometry of the sliding surface. The classical Newton method is used herein to optimize the function value.

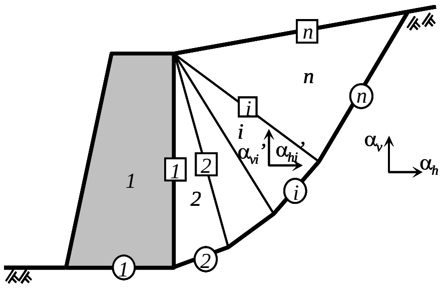


Figure 3. *Active failure (a)*

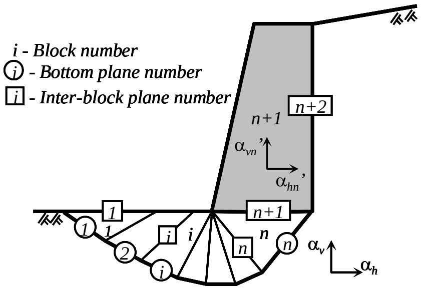


Figure 4. *Foundation – like failure (b)*

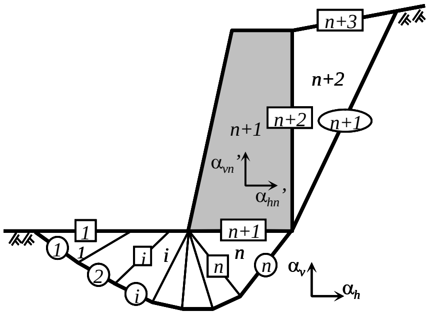


Figure 5. *Slope – like failure (c)*

IV. NUMERICAL EXAMPLE

A strip retaining wall with a mass of 25t, a height of 5m, and a width of 3m is considered. The frictional angle of back surface of the wall is 11^o. The backfill and soil base have the parameters as $\phi = 32^o$, $c = 0.11 tf/m^2$, $\gamma = 1.6 tf/m^3$. A sinusoidal wave is used as the input acceleration, which has the frequency of 2Hz, the amplitude of horizontal component is $6 m/s^2$, the vertical component equals zero. For the simplicity, the dilatancy angle is considered to be zero, and the surface of the backfill is horizontal. Four cases of analysis were carried out: case a1 - active failure mode was taken with the frictional angle of the bottom plane of the wall, δ, is 17^o; case a2 - active failure mode was taken with the frictional angle of the bottom plane of the wall is 32^o; case b - foundation-like failure mode was taken; and case c - slope-like failure mode was taken.

The results of analyses are presented in figures 6 and 7. It can be seen from the figure 6 that the solutions of the active failure mode are very different with the change of δ. When $\delta = 17^o$, the active failure occurs earlier than foundation-like failure mode (case b) and slope-like failure (case c), and it is in opposite situation for the case $\delta = 32^o$. The graph also indicates that the foundation-like failure occurs later than slope-like failure in this analysis. As stated in [6], the figure 7 once again shows the comparison between the proposed method and Mononobe-Okabe method (M-O method) [7,8] for the dynamic earth pressures. It is clear to realize that, corresponding to the sliding process, the M-O method has overestimated the earth pressure.

V. EFFECT OF ROUGHNESS OF WALL-BOTTOM SURFACE

An investigation on the relation between the frictional angle of the wall-bottom surface and the critical acceleration, at which the sliding starts to occur, was carried out. The analysis condition is the same as the example above. The interrelation between δ and the critical accelerations corresponding to every failure mode is presented in figure 8. In this analysis, when $\delta < 25.47^o$, the failure mode likely to happen is active failure. When $\delta > 25.47^o$, the failure mode likely to happen is slope-like failure. The wall seems to be safe with foundation-like failure.

Table 1. *The number of equations and unknowns*

Equations				Unknowns			
Equilibrium conditions	*(a)*	*(b)*	*(c)*	*On bottom planes*	*(a)*	*(b)*	*(c)*
In vertical direction	*n*	*n+1*	*n+2*	Normal forces	*n*	*n*	*n+1*
In horizontal direction	*n*	*n+1*	*n+2*	Shear forces	*n*	*n*	*n+1*
Failure conditions				*On inter-block planes*			
On bottom planes	*n*	*n*	*n+1*	Normal forces	*n-1*	*n+1*	*n+1*
On inter-block planes	*n-1*	*n+1*	*n+1*	Shear forces	*n-1*	*n+1*	*n+1*
Continuity Condition of Acceleration				*Block accelerations*			
On bottom planes	*n*	*n*	*n+1*	α_{vi}'	*n*	*n+1*	*n+2*
On inter-block planes	*n-1*	*n+1*	*n+1*	α_{hi}'	*n*	*n+1*	*n+2*
Total	*6n-2*	*6n+4*	*6n+8*		*6n-2*	*6n+4*	*6n+8*

VI. EXAMPLE OF A SUPPOSED RETAINING WALL SUBJECTED TO THE NIIGATA KEN CHUETSU EARTHQUAKE - 2004

A supposed retaining structure with parameters as shown in Table 2 subjected to the Niigata Ken Chuetsu Earthquake – 2004 is considered. The cross-section of the structure is assumed to oblique at an angle of about 23° to the E-W direction. So, the horizontal component of the input acceleration is derived from E-W and N-S accelerations. The vertical component is just the U-D component of the acceleration record.

It is supposed here that sliding occurs in slope-like failure mechanism. Thus, the model and block system for the problem are presented in figure 9. The peak strength of soil is applied to the duration from the beginning of the shaking to the moment that the first sliding finishes. Then the residual strength is used for the remainder of the shaking process. The wall bottom friction angle is larger than 17°, therefore no relative sliding occurs along this plane or the wall and soil wedge right beneath the wall take the same movement.

The analysis results including accelerations of the wall and sub-base, residual displacements of the wall, and dynamic active earth pressure are presented in figure 10. In order to see clearer the sliding process, the acceleration data within the duration from 19 s to 20 s are zoomed in. The first critical state is reached at moment t = 19.19 s. The slip surface at this state is obtained and considered as the actual slip surface as shown in figure 10. This slip surface is assumed to be unchanged during sliding process. With the use of peak strength, the horizontal critical acceleration for the first sliding is greater than others. The sliding occurs a number of times during shaking process (figure 11a). Over the time of shaking, the residual displacements of the wall in horizontal and vertical directions are computed and plotted in figure 11b.

Figure 11c shows the dynamic active earth pressure obtained by the present method and the M-O method. We can see once again here that during sliding the M-O method overestimates the earth pressure. At some moments, the overestimation of the M-O method is up to 130%.

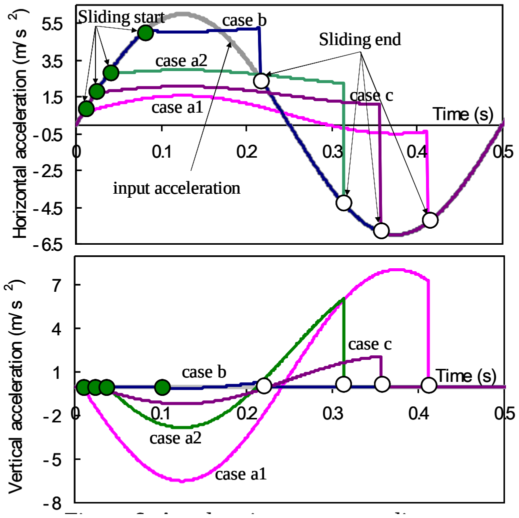


Figure 6. *Accelerations corresponding to analyzed cases*

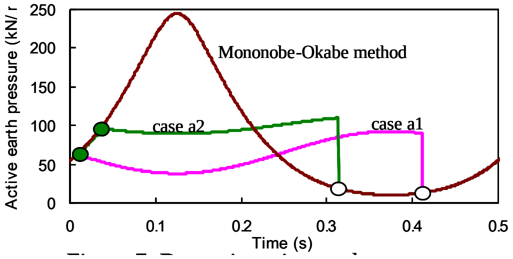


Figure 7. *Dynamic active earth pressure*

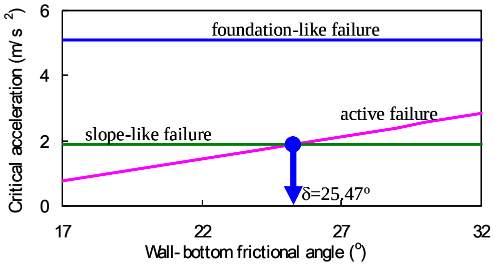


Figure 8. *Bottom surface roughness of the wall and failure modes*

Table 2. *Analysis condition for motion of a retaining structure using slope-like failure mechanism*

Wall mass, *M* (t)	20.0
Back surface friction of wall, ε (°)	10.0
Bottom surface friction of wall, δ (°)	>17.0
Soil density, ρ (t/m^3)	2.0
Internal friction angle, ϕ (°)	25.0
Cohesion, *c* (tf/m^2)	1.0

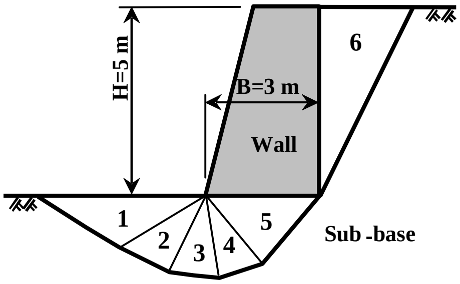


Figure 9. *Block system for analyzing motion of a retaining structure with the use of slope-like failure mechanism*

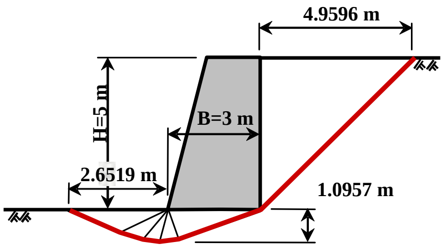
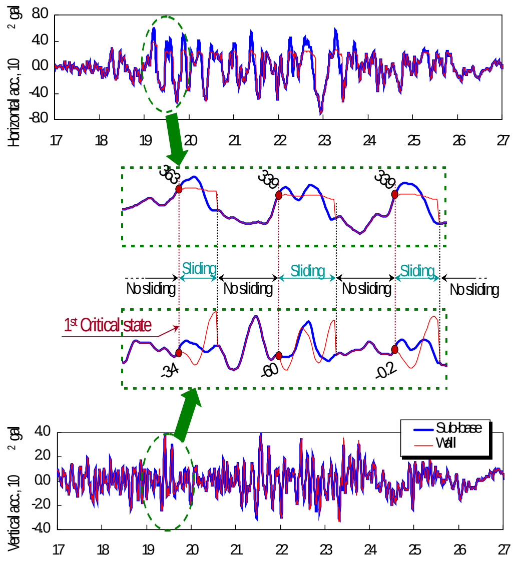
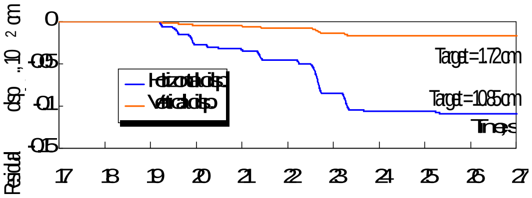


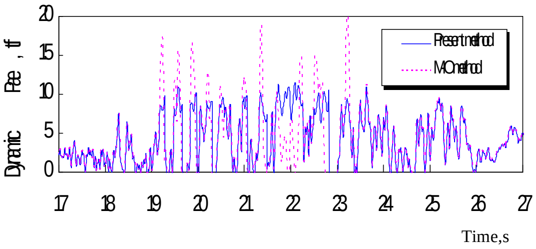
Figure 10. *Geometry of block system at the first critical state*



a. Input accelerations and sliding accelerations of the retaining wall



b. Residual displacements of the retaining wall



c. Dynamic earth pressures

Figure 11. *Results of analysis for motion of a retaining structure with the use of slope-like failure mechanism*

VII. CONCLUSIONS

The dynamic GLEM has been developed to investigate the failures of a retaining wall under seismic loadings with three mechanisms such as active failure, foundation-like failure, and slope-like failure. Not only the dynamic earth pressure but the motion of the retaining wall can be obtained also.

The foundation-like mechanism seems to be unlikely to happen. When the bottom surface roughness is low, the failure mode likely to happen is slope-like failure. When the roughness is high, the failure mode likely to happen is active failure.

When sliding has not occurred, the active earth pressures obtained by the proposed method and the M-O method are almost the same. During sliding process, the M-O method seems to overestimate the active earth pressure in comparison with the proposed method.

VIII. ACKNOWLEDGMENTS

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