AN ENERGY - EFFICIENT MODEL INTEGRATED TIME TABLE OPTIMIZATION AND REGENERATE BRAKING

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Summary: *The energy efficient of railways is paid more and more attention than before. Due to the operation characteristics of high capacity and high frequency, subway transportation is still one of the most energy-intensive industries in the world.Trainenergy-efficient operation consists of time table optimization and utilization of regenerate energy. The former finds out the proper fleet size to minimum the energy consumption, the latter synchronizes the accelerating and braking actions of trains to maximize the utilizationof regenerative energy. This paper formulates an integrated model which combines the time table optimization and utilization of regenerate energy. We design a genetic algorithm to solve the model and find out the proper fleet size for metro system.*

Keyword: *Metro system, Energy-efficient, Time table optimization, Utilization of regenerate energy.*

I. INTRODUCTION

Metro systems are electric passenger railways in urban area, which ensure passenger safety.Under the global warming infographic, the metro system is the most ecopublic transportation because of its high energyefficiency. Due to the fixed timetableandthe steady speed of the metro systems, people can choosetheir trip more flexible and be more comfortable during the trip. Because of using the electric energy, the exhaust emissions of trains are almost zero. However, due to the characteristics ofhigh capacity and high-frequencyoperation, subway transportation is stillone of the most energy-intensive industries in the world.

Metro train energy-efficient operation consists of two parts. Firstly, meeting the passengers'critical demand, the metro system timetableshould minimize the energy consumption. The cycle timeis influenced by the fleet size and interstations' trip time. The driving strategy duringinterstations is also an important factor of energy-efficient operation.Secondly,synchronizing the accelerating times and braking times of the trains which

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serviced by the same substations can improve the utilization of regenerative braking. The recovery energy from the brakingtrain can simultaneously be used by the accelerating train which means the braking energy can be used sufficiently.

The total energy consumed by metrotrains is decided by fleet size. As we all know, more trains operating at the same time, more energy will be consumed. We normallyconsider the headway is constant during a specified time, e.g. the peak-hour orthe off-peak-hour. The cycle time and fleet size is directly proportional, which means the cycle time will be longer when we choose the bigger fleet size due to the constant headway.Trains can have more time to coast during the interstations instead of high-frequency accelerating and braking, which can save lots of energy. We should choose the proper fleet size to realize the least energy consumption in the metro system.

Metro system bring in the measures that increasing the utilization of regenerative energy to save the energy. The operator in the subway system adjusts the number of service trains, travel time atsections and dwell time at stations to maximize the synchronization time between accelerating trains and braking trainssuch that the regenerative energy from braking trains can be effectively utilized by the accelerating trains.The net energy consumption is the difference between the tractive energy consumption and the utilization of regenerativeenergy, which absorbed by trains from the substations.

In order to achieve a better performance on energy saving, this paper proposes an integrated energy-efficient operationmodel to jointly timetable optimization and improvementof regenerativeenergy.Our goal is to find out the proper fleet size meeting passengers’ demand can cost least energy consumption with the view of improving utilization of regenerate energy.

II. LITERATURE REVIEW

Train operation strategy, consisting of the timetable and utilization of regenerative braking, has a great influence on the amount of energy consumption.

Metro systems have timetables, which force trains to follow each other with a fixed headway, and takethe same dwell time at stations and travel time at sections. Voorhoeve[1]first considered a Periodic Event SchedulingProblem (PESP) based model for the cyclic timetable problem.Howlett[2]et al. providedan analytical method for the problem with more than one steepslope, and a local optimization principle was used to solvethe energy-efficient driving strategy for each part of the route.Li[3] et al. studied the optimal driving strategy with an energyconstraint since the minimum energy consumption uniquelycorresponds to the given trip time.

Although regenerative braking can recover about 40% ofthe tractive energy, little work observed in the literature studies its efficient utilization method. Pena-Alcaraz[4] et al. designed a mathematical optimization model to synchronizethe braking trains with the accelerating trains for improving the utilization of regenerative energy. David[5] et al.formulated an optimization model to maximize the utilization of regenerative energy by subtly modifying dwell timefor trains at stations. Yang[6] et al. proposed a train cooperative scheduling rule to synchronize the accelerating and braking actions of successive trains.

A set of studies have been focused on the train energy-efficient operation, but very few of them have considered thecombination of timetable optimization and improvement of regenerate energy. In order to achieve a better performance on energysaving, this paper proposes an optimization model and designs a genetic algorithm to find out the proper fleet size which can save the most energy in the subway system. At the mean time the timetable also can meet most passengers' demand.

III. MODEL FORMULATION

For better understanding of this paper, the parameters are introduced firstly:

Table 1. *The parameters in model*

Indices andParameters		$\bar{u}_n$	The maximum dwell time at station n
N	Number of stations	$\underline{u}_n$	The minimum dwell time at station n
J	Number of substations	τ	The turnaround time at turnaround station
ϑ	Conversion factor from kinetic to electricity energy	$\phi(n, j)$	If the section between stations nand n+1belongto the j substation, $\phi(n, j) = 1$; otherwise, $\phi(n, j) = 0$.
α	The maximum tractive force per unit mass for trains	**Decision variables**	
β	The maximum braking force per unit mass for trains	I	Number of trains
γ	The running resistance per unit mass for trains	h	Headway
D	Passage demand	C	Cycle time
M	Train mass	a_n	Arrival time at n station for the first train
m_n	The distance between n station and n+1 station	d_n	Departure time at n station for the first train
L_n	The limit speed between station n and station n+1	**Intermediate variable**	
S_n	Top running speed between n station and n+1 station	a_n^-	Beginning time of the braking phase of the first train beforearrival at station n
S_n^+	The train speed at the brake point between n station and n+1 station	d_n^+	Ending time of the traction phase of the first train after departurefrom station n

Model assumption

Firstly, we assume that the conversion factor from kinetic to electricity energy ϑ, the turnaround time at turnaround station τ, the train mass M, the maximum tractive force per unit mass for trains α, the maximum braking force per unit mass for trains β and the running resistance per unit mass for trains γ are constant value. We ignore the slope and curvature of rail which may have little influence on our optimization model.

Secondly, we assemble the turnaround stationsand stationsfrom the up direction and down direction as a cycle which means every station except turnaround stations will be counted twice. The serial numbers of stations are showed in fig 1.

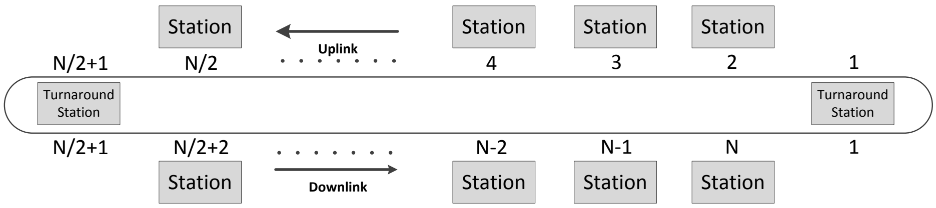

Fig 1. *The serial numbers of stations*

Energy consumption

During a period time, e.g. the off-peak-hours, the passage demand D is constant. The passages' demand decides the trains' headway h, which is constant too. The h is decided by the cycle time Cand the scale of trains I:

$$h = C / I \tag{3.1}$$

According to the optimal train control theory, the train energy-efficient movement at each segment consists of maximum acceleration, cruising, coasting,and maximum braking. However, for trains with short travel distance, such as metro trains, the energy-efficient movementonly contains accelerating, coasting and braking phases.

The speed profile of the first train described as follows:

$$v_{1n}(t) = \begin{cases} \alpha(t - d_n), & \text{if } d_n \le t \le d_n^+ \\ S_n - \gamma(t - d_n^+), & \text{if } d_n^+ \le t \le a_{n+1}^- \\ S_n - \gamma(a_{n+1}^- - d_n^+) - \beta(t - a_{n+1}^-), & \text{if } a_{n+1}^- \le t \le a_{n+1} \end{cases} \tag{3.2}$$

The first row shows that when the train depart from a station, the train speed increase with the acceleration rate α in the accelerating phase. The second row shows the train coasts without any accelerate force or braking force, the speed decrease with the running resistance γ. The last row shows when the train in the braking phase, the train speed decrease with the braking rate β.

Obviously, we can calculate the intermediate variable by using the speed-time equations:

$$\begin{cases} d_n^+ = d_n + \dfrac{S_n}{\alpha} \\ S_n = \beta(a_{n+1} - a_{n+1}^-) + \gamma(a_{n+1}^- - d_n^+) \end{cases} \quad (3.3)$$

The first row shows the train accelerates from 0 to S_n in the accelerating phases when train departs from nstation. The second row shows the train speed decrease from S_n to 0 in the coasting and braking phases before trainarrivals to n+1 station.

And according the speed-distance function, we can get the top running speed S_n between n station and n+1 station andthe train speed S_n^+ at the brake point between n station and n+1 station:

$$\begin{cases} S_n^+ = S_n - \gamma(a_{n+1}^- - d_n^+) \\ \dfrac{1}{2}\alpha(\dfrac{S_n}{\alpha})^2 + \dfrac{1}{2}\beta(\dfrac{S_n^+}{\beta})^2 + \dfrac{1}{2}(\dfrac{S_n^2 - S_n^{+2}}{\gamma}) = m_n \end{cases} \quad (3.4)$$

First row shows that the train coasts from S_n to S_n^+ after reach the top speed at interstation.Second row shows the relation between the top speed S_n, braking speed S_n^+, interstation's distance.

We consider that the top speed S_n and braking speed S_n^+ cannot overtake the limit speed L_n.

$$S_n, S_n^+ \leq L_n \quad (3.5)$$

The maximum travel time during two successive stations should contain accelerating and coasting phase except braking phase. The minimum travel time during two successive stations should contain accelerating and braking phase except coasting phase. The top speed subject to

$$\frac{S_n^2}{2\alpha} + \frac{S_n^2}{2\beta} \leq m_n \leq \frac{S_n^2}{2\alpha} + \frac{S_n^2}{2\gamma} \quad (3.6)$$

And we can calculate the range of travel time:

$$\sqrt{\frac{2(\alpha+\beta)m_n}{\alpha\beta}} \leq d_n - a_n \leq \sqrt{\frac{2(\alpha+\gamma)m_n}{\alpha\gamma}} \quad (3.7)$$

The dwell time for trains at stations should satisfy a time window constraint according to the passenger demand and theoperation time for opening/closing screen door.

$$\begin{cases} d_n - a_n = \tau, \text{n staion is turnaround station} \\ \underline{u}_n \leq d_n - a_n \leq \bar{u}_n \text{otherwise} \end{cases} \quad (3.8)$$

Next we analyzes the total energy when the trains operating during a specified time. First, we analyze the energy consumption foraccelerating trains. For each $d_n \le t \le d_n^+$, $1 \le n \le N, 1 \le i \le I$,we get the mechanical power equationas follows

$$f_{1n}(t) = \begin{cases} M\alpha v_{1n}(t), d_n \le t \le d_n^+ \\ 0, \text{otherwise} \end{cases} \tag{3.9}$$

M is the mass of train, αis the acceleration of train, $v_{1n}(t)$ is the speed profile of the first train. When train operating in the accelerating phase, the electricity is used to produce the accelerate force to increase train's speed. Otherwise, when train operating in the phase of coasting and braking, the electricity is useless.

The total energy train consume during cycle time:

$$F_1 = \sum_1^N \sum_{d_n}^{d_n^+} f_{1n}(t) \tag{3.10}$$

We divide the cycle time Cinto I components equally, each components equals h, which means the cycle time C being cutted into [0, h), [h, 2h)…[(i-1)h, ih)…[(I-1)h, C). In each piece of cycle time, the whole trains operation on the same trajectory and execute the same train operation process. So we just can analyze one piece of cycle time, for example [0, h).

The energy costed by all trains during [0, h):

$$F_h = \frac{I * \sum_1^N \sum_{d_n}^{d_n^+} f_n(t)}{I} = \sum_1^N \sum_{d_n}^{d_n^+} f_n(t) \tag{3.11}$$

Now we analyze the regenerative energy. We should notice that the regenerative energy can only be absorbed by the accelerating train which positioned at the same substation.The train can generate the regenerative energy only in baking phase.

$$g_{1n}(t) = \begin{cases} 0, \text{otherwise} \\ M\beta v_{1n}(t)\vartheta, a_{n+1}^- \le t \le a_{n+1} \end{cases} \tag{3.12}$$

The first row shows when train operating at the phases of accelerating and coasting, it cannot generate the regenerative energy. The second rows show when train operating at the phase of braking, the train can generate regenerative energy to other accelerating train. β is the maximum braking force per unit mass, ϑ isconversion factor from kinetic to electricity energy.

The general equations for train iduring [0, h) as follows:

$$g_{in}(t) = \begin{cases} 0, \text{otherwise} \\ M\beta v_{1n}\left[t + (i-1)h\right]\vartheta, a_{n+1}^- \le t + (i-1)h \le a_{n+1} \end{cases} \tag{3.13}$$

For each substation j:

$$G_j(t) = \min\left\{\sum_1^I f_{1n}[t+(i-1)h]\phi(n,j), \sum_1^I g_{1n}[t+(i-1)h)]\phi(n,j)\right\}, t \in [0,\ h) \quad (3.14)$$

That equation above shows that if there other accelerating train in the same substation j, the regenerative energy can be used by them, otherwise it can be wasted.

Integrating the equations above, we can get the regenerate energy during a period time, for example [0, h):

$$G_h = \sum_1^J \sum_0^h G_j(t) \quad (3.15)$$

So the total energy consumption during the period time of [0, *h*):

$$E_h = F_h - G_h \quad (3.16)$$

Optimization model

Based on the above analysis, we propose the following integrated energy-efficient operation model.

$$\begin{cases} \min\ E_h \\ \text{s.t. Constraints}(3.1),(3.5),(3.6),(3.7),(3.8) \end{cases}$$

IV. GENETIC ALGORITHM

In this section, wedesign a genetic algorithm to solve the integrated energy-efficient operation optimization model.

Representation structure

We set headway has constant value, like 90s. In this paper, a chromosome is definedas a N×2 - dimensional matrix X, in which X(n,1) = a_n, X(n,2) = d_n, for $1 \le n \le N$.

Initialization

For each n, X(n,1), X(n,2) should satisfy the constrains mentioned above.

We use the objective function in the model as the evaluationfunction:

$$Eval = F_h - G_h$$

Note that the minimum value of Evalmeans the bestindividual.

Selection process

The selection of chromosomes isdone by spinning the roulette wheel, which is a fitnessproportionalselection. Continuing this process pop_sizetimes, we can get the next generation. Without loss of generality,assume that the chromosomes have been ordered according to the evaluation function values.

Crossover operation

Crossover is one of the mainly used operations for generating a second population. Without loss of generality, we assume that chromosomes X_1,X_2, … ,X_C are selected and C is an even number.Divide them into the following pairs:

(X_1,X_2), (X_3,X_4), …, ($X_{C-1,}$ X_C).

We illustrate the crossover operation by parents(X_1,X_2). Denote the children as $Y_1$and Y_2.

$Y_1(n,1) = (X_1(n,1) + X_2(n,1))/2;$

$Y_1(n,2) = Y_1(n,1) + (X_1(n,2) - X_1(n,1) + X_2(n,2) - X_2(n,1))/2;$

$Y_2(n,2) = (X_1(n,2) + X_2(n,2))/2;$

$Y_2(n,1) = Y_2(n,2) - (X_1(n,2) - X_1(n,1) + X_2(n,2) - X_2(n,1))/2.$

Then we select the best two chromosomesfrom the parents and children to replace the parents.

Mutation operation

For each selected parent X, we randomly generate a dwell time u from [$\overline{u}_n$, $\underline{u}_n$], and we set:

X (n,1) = X (n,1) - (X (n,2) - X (n,1)-u);

X (n,2)= X (n,2) - (X (n,2) - X (n,1)-u).

General procedure

Following evaluation, selection, crossover and mutation operations, a new population of chromosomes is generated. Geneticalgorithm will terminate after a given number of iterations of the above steps. We now summarize the general procedurefor genetic algorithm as follows.

V. EXPERIMENTAL

The Yizhuang line is 23.23 km long with 14 stations, including 6 underground and 8 on the surface. Trainsrunning between Songjiazhuang station and Yizhuang station are powered by 6 substations.Parameters are listed as follows:conversion factor from kinetic energy to electricity $\vartheta = 0.6$, acceleration rate at accelerating phase $\alpha = 0.8 m/s^2$,deceleration rate at coasting phase $\gamma = 0.02 m/s^2$, deceleration rate at braking phase $\beta = 0.4 m/s^2$, train mass M = 287,080 kg, turnaround time $\tau = 180$ s.We set pop-size = 30, and max-generation = 50.

Table 2. *The substation distribution*

J	Interstation(S = Station)	J	Interstation(S = Station)
1	S1~ S2, S2~ S3	4	S8~ S9, S9~ S10
2	S3~ S4, S4~ S5	5	S10~ S11, S11~ S12
3	S5~ S6, S6~ S7, S7~ S8	6	S12~ S13, S13~ S14

Table 3. The position of station as follows

Station	Position	Station	Position
Songjiazhuang	0	Rongjing	12065
Xiaocun	2631	Rongchang	13419
Xiaohongmen	3906	Tongjinan	15757
Jiugong	6272	Jinghai	18022
Yizhuangqiao	8254	Ciqunan	20108
Wenhuayuan	9247	Ciqu	21394
Wanyuan	10785	Yizhuang	22728

We set headway as 90s and $I \in [45,60]$. We solve the energy-efficient model by genetic algorithm.We picked the specified I and after 50 iterations, the energy consumption of each fleet size calculated during a period time of [0, h). According to the calculation results, we find that if we choose the 52 trains operating at the same time, we can get the minimum energy consumption E_h. Comparing with the actual operating situation,the optimal fleet size of trains increased from 50 to 52, the cycle time increased from 4500 s to 4680 s,which canreduce the tractive energy consumption. The optimal energy consumption is 280 KW during the period time of [0, h), comparing with the actual energy consumption 313KW, the energy conservation rate is 10.54%.

VI. CONCLUSION

This paper formulates an integrated model which combines the time table optimization and utilization of regenerate energy. And we design a genetic algorithm to solve the model and find the most proper fleet size for metro system. We present an experiment based on the operation data of Beijing Metro Yizhuang Line of China. We conclude that when metro systems operating 52 trains at the same time, the total energy consumption is the minimum.

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