INVESTIGATION ON THE CHANGE OF COEFFICIENT DISTRIBUTIONS IN APPLICATION OF ARIMA MODEL FOR LONG-TERM DATA PROCESSING OF A NUMERICAL CABLE-STAYED BRIDGE

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Summary: *This paper focuses on data processing of long-term monitoring to assess structural conditions of a cable-stayed bridge using finite element model under the effects of temperature changes considering global deformation. A general cable-stayed bridge model was built to study both in cases of normal condition and structural condition changes. Under the changing of whole input temperature data, the time-series displacements were acquired to analyze by applying the Autoregressive Integrated Moving Average (ARIMA) model. The extracted AR-MA coefficients were figured out to show the trend in distributions. The results showed that even though the trend in distributions of coefficients did not change so much when the structural conditions change, but it can be recognized. Thus, the ARIMA model is the useful way which can be applied to process the long-term monitoring data of the cable-stayed bridges.*

Keyword: *Cable-stayed bridges, time-series analysis, global deformation, long-term data, ARIMA model.*

I.INTRODUCTION

Data processing is a vital step to assess structural conditions in structural health monitoring (SHM) of civil infrastructures as well as for the large-scaled structures, such as long-span bridges. There are two kinds of SHM data which are short-term and long-term data. The long-term data are usually very huge, therefore they cause many difficulties in storage, management and how to assess the structural conditions from them. There are two main issues in long-term data processing that need to be studied; firstly, how to convert large amount of data into usable information to assess the structural conditions; and how to detect the structural condition changes during their operation due to the effects of wind, air temperature change, loading etc. [1]. Some researches in the world applied the time-series analysis for the SHM data processing such as using the Autoregressive (AR) model, Autoregressive with exogenous input (ARX) model, or Autoregressive Moving Average model (ARMA) model etc [2-4]. However, those papers only studied on how to process the short-term data acquired from local interested positions of structures. Additionally, some researchers mentioned that the long-term deformations are irrecoverable or periodic; they are caused by the foundation settlement, the creep, the temperature effects, loading etc [5]. Therefore, those effects need to be considered while processing the long-term monitoring data.

In Vietnam, the SHM has been applied for the long-span bridges such as Bai Chay bridge, Can Tho bridge – the longest cable stayed bridge in the South East Asia, Nhat Tan cable-stayed bridge and so on. However, the data processing issue is still a big challenge for researchers or management staffs during the SHM system operations. Therefore, there are some shortcomings in applications of SHM system to assess structural conditions of bridges in Vietnam. Otherwise, The Global Positioning System (GPS) technology has been successfully used to measure displacements of oscillating flexible civil engineering structures, such as suspension bridges and high-rise buildings. By its application, the global deformation of bridges can be analyzed due to the effects of environment factors. In the previous study [1], an ARIMA model was applied for real GPS long-term data to show the trend in distribution of AR-MA coefficients. However, it was difficult to realize the structural condition changes from the distribution of AR-MA coefficients. This paper focuses on long-term data processing to assess the structural conditions of the finite element model (FEM) of a general cable-stayed bridge under the effects of temperature changes considering global deformation. In this study, a general cable-stayed bridge model was built to study in both cases of normal condition and structural condition changes. Under the changing of whole input temperature data, the time-series displacements were acquired to analyze by applying the Autoregressive Integrated Moving Average (ARIMA) model. The extracted AR-MA coefficients were figured out to show the trend in distributions. Then, the results of study show the ability of ARIMA application in the long-term monitoring data.

II. DESCRIPTION OF NUMERICAL CABLE-STAYED BRIDGE MODEL

A FEM of cable-stayed bridge is used to analyze in normal condition by using the finite element which is 420m of total length, the main span is 220m; the width of the bridge girder is 15.6m of two lanes; two towers with overall height of 90m support the structure by two planes of cables in fan type arrangement. The general layout of the cable-stayed bridge model is shown in Figure 1. The material properties and sectional properties of structural elements were referred as figured out in Table 1 and Table 2 [6].

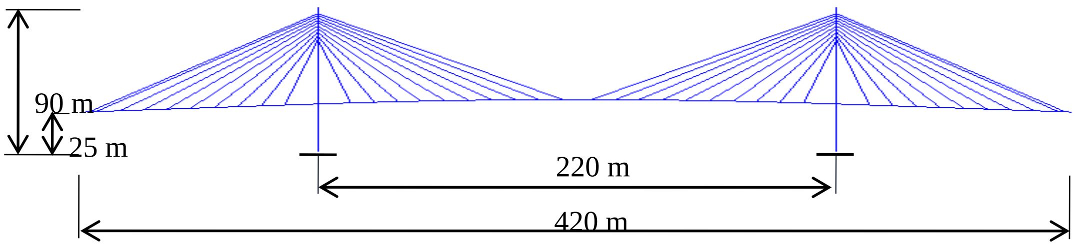


Figure 1. *General layout of the model*

Table 1. *Material properties of the model*

Material ID	Name	Modulus of Elasticity (kN/m^2)	Poisson's ratio	Weight Density (kN/m^3)
1	Cable	1.9613×10^8	0.3	77.09
2	Girder	1.9995×10^8	0.3	77.09
3	Pylon	2.78×10^7	0.2	23.56
4	CBeam-Girder	1.9995×10^8	0.3	77.09
5	CBeam-Pylon	2.78×10^7	0.2	23.56

__Table 2.__ Section properties of the model

Section ID	Name	Area (m^2)	I_{xx} (m^4)	I_{yy} (m^4)	I_{zz} (m^4)
1	Cable	0.0052	0.0	0.0	0.0
2	Girder	0.3092	0.007	0.1577	4.7620
3	Pylon	9.2	19.51	25.567	8.123
4	CBeam-Girder	0.0499	0.0031	0.0447	0.1331
5	CBeam-Pylon	7.2	15.79	14.472	7.992

In numerical analysis, the bearings are generated at two towers and two end piers of the model. The stiffness values of bearing properties are approximately 2.109 kN/m in the vertical direction (almost fixed state), and 7.105 kN/m in both longitudinal and lateral directions. In addition, the boundary supports for the analytical model are as follows:

- Tower base, Pier base: Fixed condition at (Dx, Dy, Dz, Rx, Ry, Rz)

- Connections between Main Girders and Bearings: Rigid links are fixed at (Dx, Dy, Dz, Rx, Ry, Rz).

Considering the global deformation of numerical analysis, there are four interested points extracted for analysis that are located on the top of two towers, at center of main span, and quarter center span.

III. TIME-SERIES DATA ACQUISITION

The purpose here is to assess the structural condition changes due to effects of whole temperature changes considering the global deformation behaviors. Therefore, temperature changes are used as input force set into the whole model, the displacements of four interest points under the effects of input temperature data was then acquired. This process of data acquisition is applied for all cases of study even in the normal condition and the structural condition changes.

3.1. Input temperature data and output time-series displacement data

Here, the ten-minute average temperature data of one day in sinusoidal type were generated with the amplitude of 2 degree as shown in equation 1. The displacements of all nodes in both cases: normal condition and structural condition change are then acquired under the effect of temperature changes. In the case of the structural condition change, the typical bearing properties of boundary conditions as well as the tension of cable are selected to change the values. In detail, the bearing properties at the right tower are changed to nearly fix conditions by increasing the value of stiffness in direction along to longitudinal direction and the value of stiffness rotation along to lateral direction. In addition, the longest cable near the right tower is selected to be reducedin tension of 10%. The case studies of the structural condition changes are figured out in the Figure 3.

$$temp = A\sin\left(\frac{2\pi}{T}t\right) + 27 \tag{1}$$

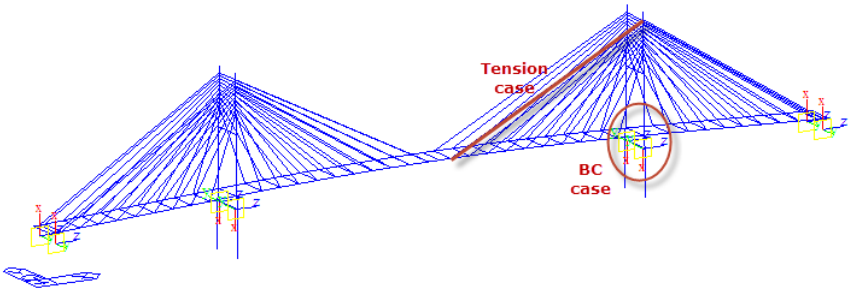
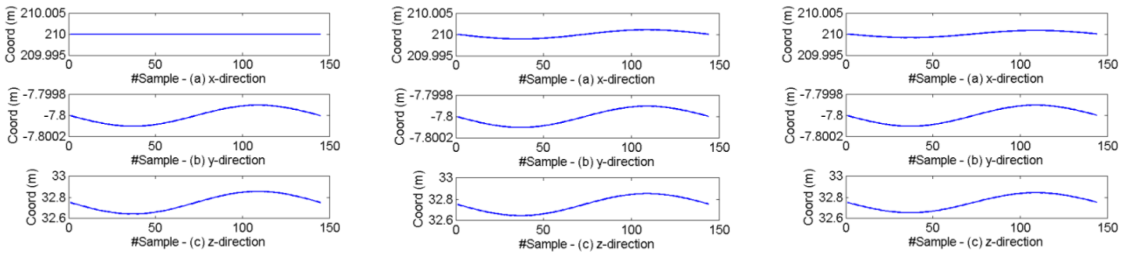


Figure 3. *Layout of structural condition change*

The output time-series displacement data in one day of center span point under the effects of input temperature change are taken to be shown in the different cases (Figure 4).



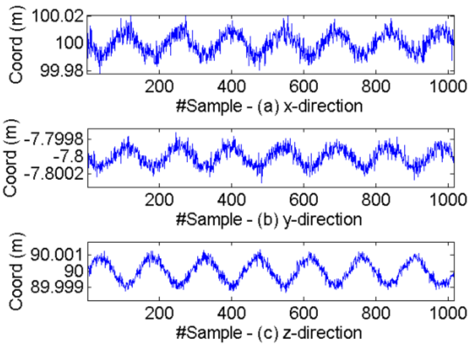
(a) Normal condition (b) Tension change (-10%) (c) Boundary condition change

Figure 4. *The output time-series displacement data of center span point*

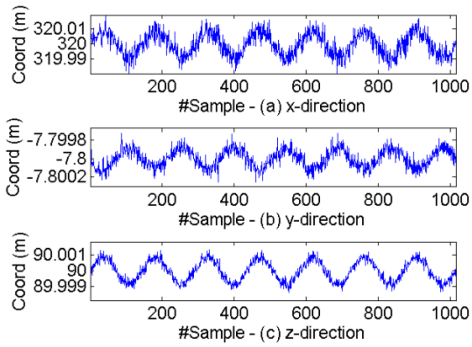
There are some remarks which could be drawn from all the plots. First of all, there is completely a periodic in the output time-series displacements due to the periodic of the input temperature data. The displacement range along the*x*-direction (longitudinal direction) in all cases of the tower points is much larger than other directions. Therefore, the *x*- direction of tower points is the significant direction of the numerical model. In addition, the displacement range along the*z*-direction in all cases of the girder points is much larger than other directions (*x*- and *y*-directions), and the vertical direction of girder points is also the significant direction (Figure 4). Otherwise, there is no deflection in the*x*-directionof center span point in the case of normal condition (Fig. 4 (a)), because the analytical model is completely symmetric model, thus the center point does not move along the*x*-direction. However, the deflection along the*x*-direction is larger than *y*-direction in bothboundary condition and cable tension changes (Fig. 4 (b, c)). It can be explained that the structural conditions were changed at non-symmetric locations. The discussions in the center span point are also the same trends in other interested points. It can be concluded that, the displacements of four interested points along to the significant directions are changed when the structural conditions of model change.

3.2. Adding white noise into output time-series displacement data

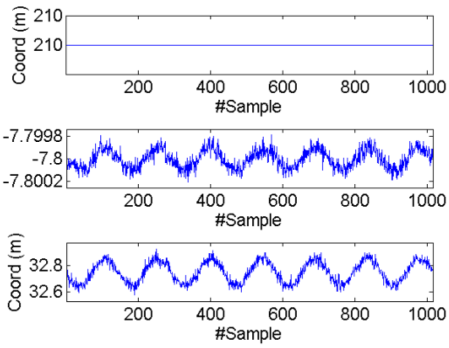
The output acquired time-series data are then added the random white noise. The range of random noise is referred from the real cable-stayed bridge case [1] where the ratio of the error and the displacement ranges in horizontal plan (*x*- and *y*-direction) is around 25% and this ratio along to vertical direction is around 12%. Therefore, the ranges of adding random noise are based on those ratios. Consequently, the one week sample of numerical time-series datawith random noise in the normal condition are shown in the Figure 5.Considering the global deformation behavior of bridge model as referred in the results of the real cable-stayed bridge, the significant directions of the bridge model are *x*-direction of tower points, *z*-direction of girder points, and the *x*-direction of the quarter span point, respectively. These significant directions are used for analysis in the next step of study.



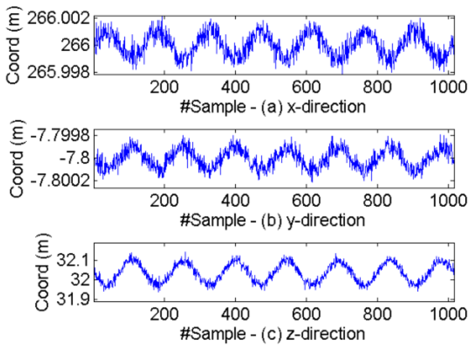
(a) The left tower point



(b) The right tower point



(c) The center span point



(d) The quarter span point

Figure 5. *The one week data of the interested points in normal condition*

IV. APPLICATION OF TIME-SERIES AUTOREGRESSIVE INTEGRATED MOVING AVERAGE (ARIMA) MODEL FOR FEATURE EXTRACTION

The numerical time-series data in the normal condition case and the cases of structural condition change were then applied the ARIMA model to extract the AR-MA coefficients. The ability of using AR-MA coefficients to detect the structural condition changes is discussed in this chapter. Considering the global deformation behavior of numerical model, the *x*-direction (longitudinal) of the left and right tower points (#Pt1 and #Pt4), quarter span point (#Pt3), and the z-direction (vertical) of the center and quarter center points (#Pt2 and #Pt3) were taken for analysis in this step of study.

4.1. General of ARIMA model and its application

The ARIMA model is a statistical model to describe non-stationary time-series [7,8]. The model is generally described as ARIMA(p, d, q),where p, d and q are the orders of the model, and it is defined as:

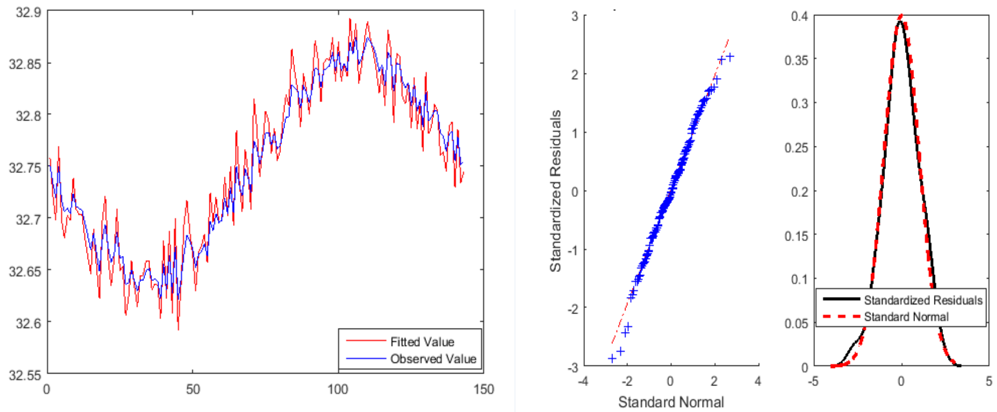
$$\left(1-\phi_1 B-\phi_2 B^2-\cdots-\phi_p B^p\right)\left(1-B\right)^d y_t = c+\left(1-\psi_1 B-\psi_2 B^2-\cdots-\psi_q B^q\right)\varepsilon_t \qquad (2)$$

Where y_t is the$t$-th component of the target time-series vector**y**, and$\varepsilon_t$is the white noise error term. ϕ_i (i =1-p) and ψ_j (j =1-q) are coefficients of the autoregressive term and the moving average term, respectively, and $B$is the backshift operator, which is defined as:

$$B^k y_t = y_{t-k}\,; \qquad (3)$$

Therefore, $(1\text{-}B)^d$ in Eq.(2) indicates the d-th order difference of the original time-series. By taking the difference, the trend in the time-series can be removed, and the non-stationary process can be transferred to the stationary process. The values of orders p, d, q are determined by applying the Autocorrelation function (ACF), Partial Autocorrelation function (PACF), and Bayesian Information Criterion (BIC). In this study, the appropriate orders were defined as (1, 1, 1). Then, the length of time-series numerical data is taken in 20 days, and the data is divided day by day for applying the ARIMA(1,1,1) model.

Figure 6 shows the example results of one-day ARIMA model performance that the time-series from the estimated ARIMA(1,1,1) model and the original one are overlaid in Figure 6(a), and the Figure 6(b) is the standardized distribution of the residual errors with the standardized normal distribution. It can be seen that, the estimated time-series data and the original one are fitted quite good, and the distribution of the residual error looks like the white noise distribution. To ensure the normal distribution of the residual error, the Ljung-Box Q-test (LBQ-test) [9] was also conducted to check whether the residual error distribution showed the white noise process with the normal distribution. The null hypothesis here was "the residual was the white noise". In the results, the statistical possibility p-values at lags (3, 6, 9) were (0.1503, 0.2767, 0.0808) that are all larger than the 5% significant level, and the test statistic values are calculated at (2.0695, 5.1049, 12.6617), which are all smaller than the critical values at 5% significant level (3.8415, 9.4877, 14.0671). It means that the null hypothesis was not rejected. The residuals are thus identified as the white noise process. We also checked those performances in the estimations to other one-day time-series and confirmed that the estimations were conducted with almost the same and appropriate accuracies.

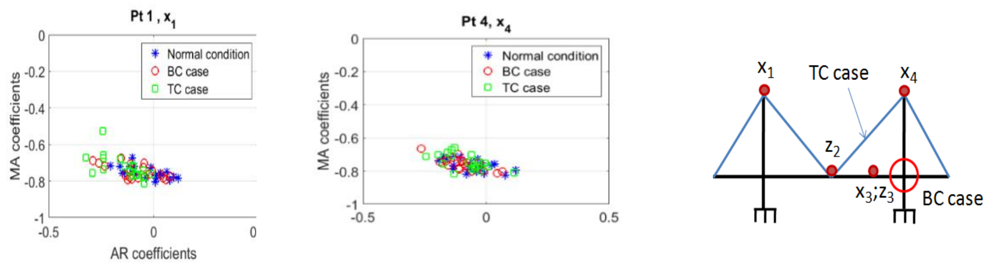


(a) Overlay of estimated and numerical series *(b) Distribution of the residual error*

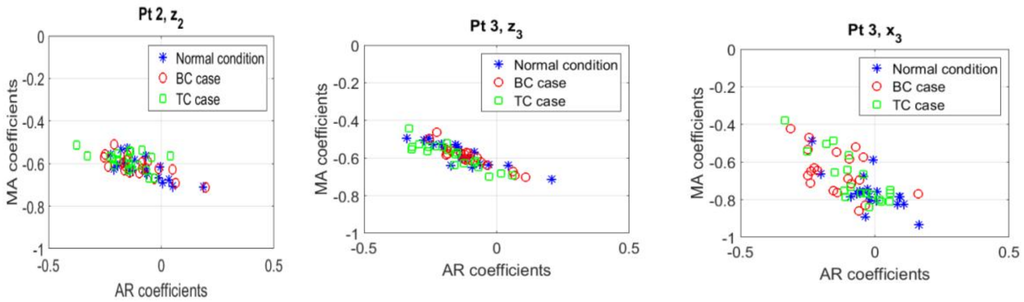
Figure 6. *Estimated results of one ARIMA (1,1,1) model*

4.2. AR-MA coefficient distributions analysis

In this chapter, the AR-MA coefficients were extracted from both cases of normal condition and structural condition changes (BC & TC cases), and all coefficients were then overlaid in Figure 7.In all plots, the star symbol denotes the AR-MA coefficients of model in the normal condition, and the circle, square symbols showsthe AR-MA coefficients in the cases of boundary condition and cable tension changes, respectively. The first row plots the coefficient distributions of the tower points (#Pt1 and #Pt4) along the*x*-direction, and the second row shows the coefficient distributions of the girder points (#Pt2 and #Pt3) along the*z*-direction as well as the coefficient plot of the quarter span points (#Pt3) along the*x*-direction.



(a) #Pt1 – x direction *(b) #Pt4 – x direction*



(c) #Pt2 – z direction *(d) #Pt3 – z direction* *(e) #Pt3– x direction*

Figure 7. *Overlaid plot of AR-MA coefficients*

Firstly, in both cases of structural condition changes, there is a different trend in the distribution of AR-MA coefficients to compare with the normal condition. However, this kind of trend is not so clear in almost significant directions except the x-direction of the quarter span point, even though there are some scatter coefficients. In the plot of the quarter span point along the longitudinal direction, there is clearly difference between the distributions of coefficients in both cases. It can be explained that its location is the non-symmetric, thus the boundary condition changes can be easier to realize there by looking at the AR-MA coefficient distributions. Overall, even though there is not so clear difference in the distributions of the AR-MA coefficients when the structural condition changes, the trend in their distributions could be used to understand the structural condition change.

V. CONCLUSION

In this paper, a cable-stayed bridge model is built to investigatethe ability of using AR-MA coefficients to detect the structural condition changes under the effects of time-series temperature changes considering global deformation behaviors. The extracted AR-MA coefficients in all cases of study are figured out to show their trend in distribution. The emphasized conclusion here is that, the trend in distribution of the AR-MA coefficients could be used to realize the structural condition changes even though it is not so clear to see the changes in those distributions except the longitudinal direction of the quarter span point. Therefore, the time-series ARIMA model is the appropriate method that can be applied for the long-term monitoring data processing of cable-stayed bridges. In case of damage detection, the AR-MA coefficients may be used as features to be applied for other methods.

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