MATHEMATICAL MODEL FOR STUDYING THE STABILITY OF DYNAMIC SYSTEM

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Summary: *In this paper, the author studies the stability of the cylindrical panel structures while they are affected by aerodynamics. Through the analysis of the effect of aerodynamic on the structural by analytic methods, the author has established a basic mathematical model and using this model to study the stability behavior of the cylindrical panel structures in aerodynamic flux with a supersonics speed.*

Key words: *Aerodynamics, stability, structures, cylindrical panel.*

I. INTRODUCTION

The aerodynamic stability of the technical systems is one of top criteria in calculating the structure of structures subjected to aerodynamic load. The stability of composite structures has been studied by many scientists [1],[2],[3],[6],[7]. However, in their publications concerning the stability of these structural forms, the authors have mainly used the finite element method [1], [8] and they have considered with only special cases [4], [5].

The main content of the presentation below, we study the dynamic stability behavior of the cylindrical panel structures that are in the aerodynamic flux that moves supersonic, establishes dynamics governing equations and investigates nonlinear vibration of cylindrical panel. In the previous results, other authors only considered the individual cases associated with a specific oscillation mode. In this paper, we considered the general case with any oscillation mode. On qualitatively, by analytic method, it was explicitly established the 2^{nd} order nonlinear differential equations system which depends on geometric parameters as well as depends on the oscillator mode. By the numerical method applied to the established model, we have conducted quantitative research on the stability of the technical system.

II. CONTENT

2.1. The nonlinear flutter of structure under aerodynamic loads

Consider a cylindrical panel of thickness h, width a, length of the arc b, radius of a middle surface r Select the coordinate axis 0 xyz so that the axes 0x, 0y describe the edge a, arc b, in that sense, the (0x, 0y) is defined as the Cartesian coordinates overlaps the rectangular plane area of the panel; axis 0z describe the direction of the normal corresponds to the underside of the middle surface of the structure. The ratio between the volume of metal $V_c(z)$ and the one of ceramic $V_c(z)$ is distributed according to the law.

$$V_m(z)+V_c(z)=1,\ \ V_c(z)=\left(\frac{2z+h}{2h}\right)^k \tag{1}$$

In which, $k\geq 0$ is the volume-fraction index, z is the thickness coordinate of the cylindrical panel structure and $z\in\left[-\frac{h}{2},\frac{h}{2}\right]$. According to the mixed law, module Young $E(z)$ and mass density $\rho(z)$ of the material are expressed under the form

$$\begin{cases} E(z)=E_cV_c+E_mV_m=E_m+(E_c-E_m)\left(\dfrac{2z+h}{2h}\right)^k \\ \rho(z)=\rho_cV_c+\rho_mV_m=\rho_m+(\rho_c-\rho_m)\left(\dfrac{2z+h}{2h}\right)^k, \end{cases} \tag{2}$$

where E_m, E_c and $\rho_m(z), \rho_c(z)$ are respectively the module Young and mass densities of the metal and of the ceramic.

The displacement of a material point $M(x,y,z)$ in the cylindrical panel has moving components u, v and w in the directions of $0x, 0y$ and $0z$. The displacement in a neighborhood of the point consists of displacement parts that cause the strains and circular motion. The distorted elements are constrained by the conditions that are suitable for the change, in order to ensure the existence of continuous and monotonic solutions.

Deformations vector $\varepsilon(M)=(\varepsilon_x, \varepsilon_y, \gamma_{xy})$ at the point $M(x,y,z)$ having the distance z from the middle surface of the cylindrical panel structure are related to the displacements components u, v, w; that are presented by vector $\varepsilon^0=(\varepsilon_x^0, \varepsilon_y^0, \gamma_{xy}^0)$ and by curvature of the cylindrical panel that is bended the curvatures vector of bending cylindrical panel $\chi=(\chi_x,\chi_y,\chi_{xy})$ via the formula $\varepsilon_x=\varepsilon_x^0-z\chi_x,\ \varepsilon_y=\varepsilon_y^0-z\chi_y,\ \gamma_{xy}=\gamma_{xy}^0-2z\chi_{xy}$, where

$$\begin{cases} \varepsilon_x^0=\dfrac{\partial u}{\partial x}+\dfrac{1}{2}\left(\dfrac{\partial w}{\partial x}\right)^2,\ \chi_x=\dfrac{\partial^2 w}{\partial x^2};\ \ \varepsilon_y^0=\dfrac{\partial v}{\partial y}-\dfrac{1}{r}w+\dfrac{1}{2}\left(\dfrac{\partial w}{\partial y}\right)^2,\ \ \chi_y=\dfrac{\partial^2 w}{\partial y^2} \\ \gamma_{xy}^0=\dfrac{\partial u}{\partial y}+\dfrac{\partial v}{\partial x}+\dfrac{\partial w}{\partial x}\dfrac{\partial w}{\partial y},\ \ \chi_{xy}=\dfrac{\partial^2 w}{\partial x\,\partial y}. \end{cases}$$

The deformation compatibility equation is defined as follows

$$\frac{\partial^2\varepsilon_x^0}{\partial y^2}+\frac{\partial^2\varepsilon_y^0}{\partial x^2}-\frac{\partial^2\gamma_{xy}^0}{\partial x\,\partial y}=\left(\frac{\partial^2 w}{\partial x\,\partial y}\right)^2-\frac{\partial^2 w}{\partial x^2}\frac{\partial^2 w}{\partial y^2}-\frac{1}{r}\frac{\partial^2 w}{\partial x^2}. \tag{3}$$

Hooke’s law describes the relationship between stress and deformation of the structure by

$$\sigma_x=\frac{E(z)}{1-\nu^2}(\varepsilon_x+\nu\varepsilon_y),\ \sigma_y=\frac{E(z)}{1-\nu^2}(\varepsilon_y+\nu\varepsilon_x),\ \tau_{xy}=\frac{E(z)}{2(1+\nu)}\gamma_{xy}. \tag{4}$$

Integrate the stress-strain components and their moment according to thickness of the structure, we have identified expressions for the internal force and moment through components of the deformation and curvature of the structure.

According to the nonlinear Piston theory, an intensity of aerodynamic force affecting on the structure is defined by: $q_0 = -\gamma P_\infty\left(M\frac{\partial w}{\partial x}+\frac{1}{a_\infty}\frac{\partial w}{\partial t}\right)-\frac{\gamma(\gamma+1)}{4}P_\infty\left(M\frac{\partial w}{\partial x}+\frac{1}{a_\infty}\frac{\partial w}{\partial t}\right)^2$

in which γ is the heat capacity of gas, a_∞ is the speed of sound, P_∞ is the gas pressure without perturbation. With U is the speed of gas flow, $M := \frac{U}{a_\infty}$ is the Mach number, which characterizes the compression strength of the moving gas flow.

The motion equation of the structure according to the Love theory and the Volmir hypothesis has the following form, with $\rho_0 = \int_{-h/2}^{h/2}\rho(z)dz = \left(\rho_m + \frac{\rho_c-\rho_m}{k+1}\right)h$:

$$\frac{\partial^2 M_x}{\partial x^2}+2\frac{\partial^2 M_{xy}}{\partial x\partial y}+\frac{\partial^2 M_y}{\partial y^2}+N_x\frac{\partial^2 w}{\partial x^2}+2N_{xy}\frac{\partial^2 w}{\partial x\partial y}+N_y\frac{\partial^2 w}{\partial y^2}+\frac{N_y}{r}+q_0=\rho_0\frac{\partial^2 w}{\partial t^2}. \quad (5)$$

From (3), we transform the deformation compatibility equation into

$$\frac{1}{E_1}\Delta\Delta\varphi = -\frac{1}{r}\frac{\partial^2 w}{\partial x^2}+\left(\frac{\partial^2 w}{\partial x\partial y}\right)^2-\frac{\partial^2 w}{\partial x^2}\frac{\partial^2 w}{\partial y^2}. \quad (6)$$

From (5), the components of the internal force and moment are expressedthrough the deflection and the stress function, the equation of motion is

$$\begin{aligned}&\rho_0\frac{\partial^2 w}{\partial t^2}+\gamma P_\infty\left(M\frac{\partial w}{\partial x_1}+\frac{1}{a_\infty}\frac{\partial w}{\partial t}\right)+\frac{\gamma(\gamma+1)P_\infty}{4}\left(M\frac{\partial w}{\partial x_1}+\frac{1}{a_\infty}\frac{\partial w}{\partial t}\right)^2+\frac{E_1E_3-E_2^2}{E_1(1-\nu^2)}\Delta\Delta w+\\&+2\frac{\partial^2\varphi}{\partial x\partial y}\frac{\partial^2 w}{\partial x\partial y}-\frac{\partial^2\varphi}{\partial y^2}\frac{\partial^2 w}{\partial x^2}-\frac{\partial^2\varphi}{\partial x^2}\frac{\partial^2 w}{\partial y^2}-\frac{1}{r}\frac{\partial^2\varphi}{\partial x^2}=0.\end{aligned} \quad (7)$$

The system of equations (6) and (7) has unknown functions φ and w describing the nonlinear vibration of the cylindrical panel structure under aerodynamic force. This system is used to study the dynamic stability of structure.

2.2. The analysis of nonlinear dynamic models

With $f_1(t)$, $f_2(t)$ are time dependent total amplitudes, we will find the solutions of the equations (6) and (7) in the form

$$w=\left[f_1(t)\sin\frac{m\pi x}{a}+f_2(t)\sin\frac{(m+1)\pi x}{a}\right]\sin\frac{n\pi y}{b}, \quad (8)$$

that are satisfy boundary conditions

$$\left(w=M_x=N_y=N_{xy}\right)\Big|_{(x=0,x=a)}=0;\ \left(w=M_y=N_y=N_{xy}\right)\Big|_{(y=0,y=b)}=0 \quad (9)$$

and the initial condition

$$f_1(t=0)=f_1^{(0)},\ \dot f_1(t=0)=f_{11}^{(0)};\ f_2(t=0)=f_2^{(0)},\ \dot f_2(t=0)=f_{21}^{(0)}. \quad (10)$$

Solving the equation (6) with the formula of the equation's solution determined by (8), we find a stress function φ in the form

$$\varphi = \varphi_1 \sin\frac{m\pi x}{a}\sin\frac{n\pi y}{b} + \varphi_2 \sin\frac{(m+1)\pi x}{a}\sin\frac{n\pi y}{b}$$
$$+\varphi_3\cos\frac{2n\pi y}{b} + \varphi_4\cos\frac{2m\pi x}{a} + \varphi_5\cos\frac{2(m+1)\pi x}{a} + \varphi_6\cos\frac{(2m+1)\pi x}{a} \tag{11}$$
$$+\varphi_7\cos\frac{\pi x}{a} + \varphi_8\cos\frac{(2m+1)\pi x}{a}\cos\frac{2n\pi y}{b} + \varphi_9\cos\frac{\pi x}{a}\cos\frac{2n\pi y}{b}.$$

$$\begin{cases} \varphi_1 = \dfrac{E_1 f_1 m^2 a^2}{r\pi^2\left(m^2+n^2\lambda^2\right)},\ \varphi_2 = \dfrac{E_1 f_2 (m+1)^2 a^2}{r\pi^2\left[(m+1)^2+n^2\lambda^2\right]^2},\ \varphi_4 = \dfrac{E_1 f_1^2 n^2\lambda^2}{32m^2} \\ \varphi_3 = \dfrac{E_1\left[f_1^2 m^2 + f_2^2(m+1)^2\right]}{32n^2\lambda^2},\ \ \varphi_5 = \dfrac{E_1 f_2^2 n^2\lambda^2}{32(m+1)^2},\ \varphi_6 = \dfrac{E_1 f_1 f_2 n^2\lambda^2}{4(2m+1)^2}, \\ \varphi_7 = \dfrac{-E_1 f_1 f_2 n^2\lambda^2}{4},\ \varphi_8 = \dfrac{-E_1 f_1 f_2 n^2\lambda^2}{4\left[(2m+1)^2+4n^2\lambda^2\right]^2},\ \varphi_9 = \dfrac{E_1 f_1 f_2 n^2(2m+1)^2\lambda^2}{4\left(1+4n^2\lambda^2\right)^2}. \end{cases} \tag{12}$$

Using the Galerkin's method for the equation (7), we obtain the motion differential equations system of the structure.

- Consider the case $m := 2m_1,\ m_1 \in \mathbb{N}$, we have

$$\rho_1\frac{a^4}{\pi^4}\ddot{f}_1 + \frac{E_1E_3-E_2^2}{E_1(1-\nu^2)}\left(m^2+n^2\lambda^2\right)^2 f_1 + \frac{\gamma P_\infty a^4}{\pi^4}\left[-f_2\frac{4Mm(m+1)}{a(2m+1)}\right.$$
$$\left.+\frac{\dot{f}_1}{a_\infty}\right] + \frac{f_1 f_2^2 n^4\lambda^4 E_1}{16}\left\{4 + \frac{m^2(m+1)^2}{n^4\lambda^4} + \frac{(2m+1)^4}{(1+4n^2\lambda^2)^2} + \frac{1}{[(2m+1)^2+4n^2\lambda^2]^2}\right\}$$
$$+\frac{f_1^3E_1}{16}\left(m^4+n^4\lambda^4\right) - \frac{2f_1f_2a^2n\lambda^2\delta_n E_1}{r\pi^4}\left\{\frac{(2m+1)^2}{3\left(1+4n^2\lambda^2\right)^2} - \frac{m}{(3m+1)(m+1)}\right.$$
$$\left.-\frac{m(2m+1)^2}{3(3m+1)(m+1)\left[(2m+1)^2+4n^2\lambda^2\right]^2} + 1\right\} + \frac{16f_1f_2a^2nm^2\lambda^2\delta_n E_1}{3r\pi^4(3m+1)\left(m^2-1\right)}\times$$
$$\times\left\{\frac{m^2}{\left(m^2+n^2\lambda^2\right)^2} + \frac{(m+1)^2}{\left[(m+1)^2+n^2\lambda^2\right]^2}\right\}\left[2m^2+(m+1)^2\right] - \frac{8P_\infty\gamma(\gamma+1)a^2\delta_n}{3n\pi^4(3m+1)}\times$$
$$\times\left[M^2\frac{f_1f_2m^2(m+1)}{(m-1)} + \frac{2\dot{f}_1\dot{f}_2m^2}{\left(m^2-1\right)a_\infty^2}\right] + \frac{m^4a^4E_1f_1}{r^2\pi^4\left(m^2+n^2\lambda^2\right)^2} = 0 \tag{13}$$

$$\rho_1\frac{a^4}{\pi^4}\ddot{f}_2 + \frac{E_1E_3-E_2^2}{E_1(1-\nu^2)}[(m+1)^2+n^2\lambda^2]^2 f_2 + \frac{(m+1)^4a^4E_1f_2}{r^2\pi^4\left[(m+1)^2+n^2\lambda^2\right]^2}$$
$$+\frac{\gamma P_\infty a^4}{\pi^4}\left[f_1\frac{4Mm(m+1)}{a(2m+1)} + \frac{\dot{f}_2}{a_\infty}\right] + \frac{f_2^3E_1}{16}[(m+1)^4+n^4\lambda^4] +$$

$$\frac{f_2 f_1^2 n^4 \lambda^4 E_1}{16}\left\{4+\frac{m^2(m+1)^2}{n^4\lambda^4}+\frac{(2m+1)^4}{(1+4n^2\lambda^2)^2}+\frac{1}{[(2m+1)^2+4n^2\lambda^2]^2}\right\}$$

$$+\frac{2f_1^2 a^2 n\lambda^2\delta_n E_1}{r\pi^4(3m+1)(m^2-1)}\left[\frac{8m^4(3m^2+2m+1)}{3(m^2+n^2\lambda^2)^2}+\frac{(m+1)^2}{2}\right]+$$

$$+\frac{2f_2^2 a^2 n\lambda^2\delta_n E_1}{3r\pi^4}\left\{\frac{8(m+1)^3}{\left[(m+1)^2+n^2\lambda^2\right]^2}+\frac{1}{2(m+1)}\right\}-\frac{4\gamma(\gamma+1)P_\infty a^2\delta_n}{3n\pi^4}\times$$

$$\times\left\{M^2\left[\frac{(m^2-2m-1)f_1^2}{(m^2-1)(3m+1)}+\frac{(m+1)f_2^2}{3}\right]+\frac{2\dot{f}_1^2 m^2}{(3m+1)(m^2-1)a_\infty^2}\right\}=0 \qquad (14)$$

- Consider the case $m:=2m_1+1,\ m_1\in\mathbb{N}$, we have

$$\rho_1\frac{a^4}{\pi^4}\ddot{f}_1+\frac{E_1E_3-E_2^2}{E_1(1-\nu^2)}(m^2+n^2\lambda^2)^2 f_1+\frac{\gamma P_\infty a^4}{\pi^4}\left[-f_2\frac{4Mm(m+1)}{a(2m+1)}+\frac{\dot{f}_1}{a_\infty}\right]+$$

$$\frac{f_1 f_2^2 n^4\lambda^4 E_1}{16}\left\{4+\frac{m^2(m+1)^2}{n^4\lambda^4}+\frac{(2m+1)^4}{(1+4n^2\lambda^2)^2}+\frac{1}{[(2m+1)^2+4n^2\lambda^2]^2}\right\}+\frac{f_1^3E_1}{16}$$

$$\times(m^4+n^4\lambda^4)+\frac{2f_1^2a^2n\lambda^2\delta_n E_1}{3r\pi^4}\left[\frac{8m^3}{(m^2+n^2\lambda^2)^2}+\frac{1}{2m}\right]-\frac{2f_2^2a^2n\lambda^2\delta_n E_1}{r\pi^4(3m+2)(m+2)}\times$$

$$\times\left\{\frac{8(m+1)^4(3m^2+4m+2)}{3m\left[(m+1)^2+n^2\lambda^2\right]^2}+\frac{m}{2}\right\}+\frac{m^4a^4E_1 f_1}{r^2\pi^4(m^2+n^2\lambda^2)^2}-\frac{4\gamma(\gamma+1)P_\infty a^2\delta_n}{3n\pi^4}\times$$

$$\left\{M^2\left[\frac{f_1^2 m}{3}+\frac{f_2^2(m+1)^2(m^2+4m+2)}{m(m+2)(3m+2)}\right]+\frac{2}{a_\infty^2 m}\left[\frac{\dot{f}_1^2}{3}+\frac{\dot{f}_2^2(m+1)^2}{(m+2)(3m+2)}\right]\right\}=0 \qquad (15)$$

$$\rho_1\frac{a^4}{\pi^4}\ddot{f}_2+\frac{E_1E_3-E_2^2}{E_1(1-\nu^2)}[(m+1)^2+n^2\lambda^2]^2 f_2+\frac{(m+1)^4a^4E_1 f_2}{r^2\pi^4\left[(m+1)^2+n^2\lambda^2\right]^2}+\frac{\gamma P_\infty a^4}{\pi^4}\left[\frac{\dot{f}_2}{a_\infty}+\right.$$

$$\left.f_1\frac{4Mm(m+1)}{a(2m+1)}\right]+\frac{f_2 f_1^2 n^4\lambda^4E_1}{16}\left\{4+\frac{m^2(m+1)^2}{n^4\lambda^4}+\frac{(2m+1)^4}{(1+4n^2\lambda^2)^2}+\frac{1}{[(2m+1)^2+4n^2\lambda^2]^2}\right\}$$

$$+\frac{f_2^3E_1}{16}\left[(m+1)^4+n^4\lambda^4\right]+\frac{2f_1f_2a^2n\lambda^2\delta_n E_1}{3r\pi^4 m}\left\{\frac{(m+1)(2m+1)^2}{(m+2)(1+4n^2\lambda^2)^2}-\right.$$

$$\left.\frac{(2m+1)^2}{(3m+2)\left[(2m+1)^2+4n^2\lambda^2\right]^2}+\frac{1}{3m+2}+\frac{m+1}{m+2}\right\}-\frac{16f_1f_2a^2n\lambda^2\delta_n E_1}{3mr\pi^4(3m+2)(m+2)}\times$$

$$\times\left\{\frac{m^2}{\left(m^2+n^2\lambda^2\right)^2}+\frac{\left(m+1\right)^2}{\left[\left(m+1\right)^2+n^2\lambda^2\right]^2}\right\}\left[2\left(m+1\right)^2-m^2\right]-\frac{8\gamma\left(\gamma+1\right)P_\infty a^2\delta_n}{3n\pi^4}\times$$

$$\times\left\{M^2\frac{f_1f_2m\left(m+1\right)^2}{\left(3m+2\right)\left(m+2\right)}+\frac{1}{a_\infty^2}\left[\frac{\dot{f}_2^2}{3\left(m+1\right)}+\frac{2\dot{f}_1\dot{f}_2\left(m+1\right)^2}{m\left(3m+1\right)\left(m+2\right)}\right]\right\}=0 \qquad (16)$$

The motion differential equations system (13-16) are established by analytic method, describes flutter and the flutter frequency of the structure under aerodynamic force.

2.3. Estimate the suitability of the model

We used numerical method to research the stability of the model. By using RK4 method in Matlab and study the influence of composite elements of a material on the stability of flutter frequency by using numerical analysis. Observe oscillations of the structure with $k=1,\ a=b=1.8m,\ h=0.003m,\ R=3m.$

When $M=2.7668$, the structure is stable, the amplitude and frequency of oscillation in Figure 1 is in the form of harmonic oscillation. As the Mach number increases to 0.001 (ie increased airflow velocity 0.034m/s), the structural is unstable andthe amplitude increase quickly over time (Figure 2).

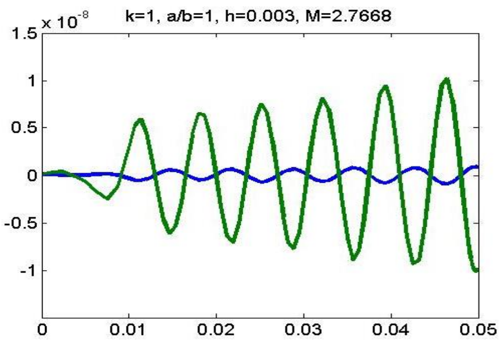

Figure 1. *The structure is stable with M = 2.7668*

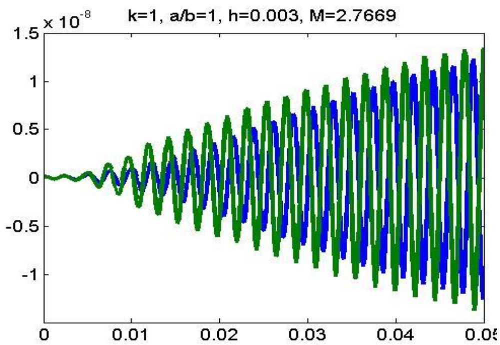

Figure 2. *The structure is unstable with M = 2.7669*

Consider the effect of the a/b ratio on the limited Mach. As this ratio increases, it decreases. This is shown in Figure 3 and Figure 4, with the ratio $a/b=2,$ we see $M=1.3118$, the structure is unstable (compared to case $a/b=1,$ as shown in Figure 2, the structure is unstable with $M=2.7669$).

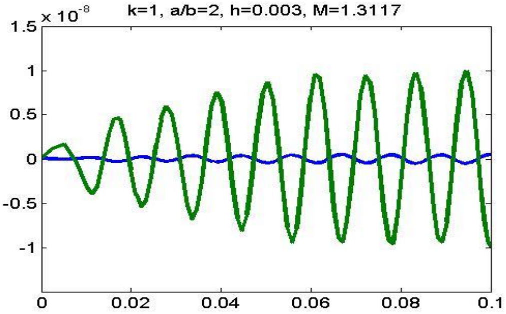

__Figure 3.__ The structure is stable with M= 1.3117

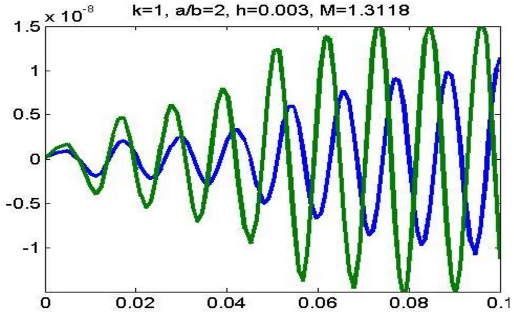

__Figure 4.__ The structure is unstable with M= 1.3118

When changing the curvature of the structure, the radius of the structure increases, the limited Mach decreases markedly. Figures 5 and 6 are vibrations of the structure with $R = 10\,m$. When $M = 1.2564$, the structure starts to become unstable (compared to the case where $R = 3m$ in Figure 2, we see that when $M = 2.7669$, the structure begins to instability).

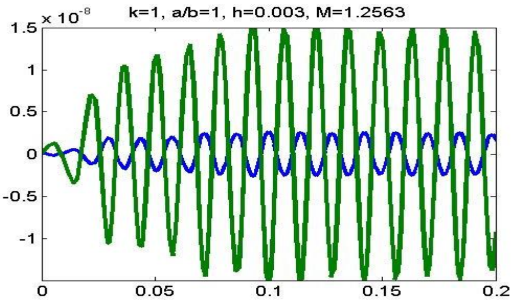

__Figure 5.__ The structure is stable with M = 1.2563

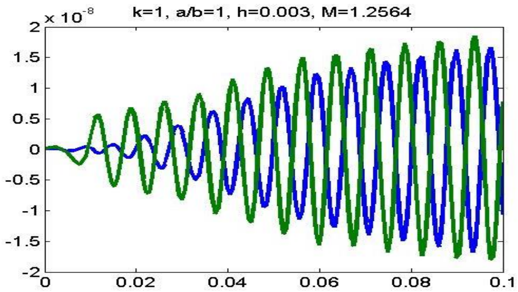

__Figure 6.__ The structure is unstable with M = 1.2564

Change the k index of component materials: When k increases, the Mach decreases. This is suitable because the elastic modulus of the metal is much smaller than that of the ceramic. Figures 7 and 8 show the dynamic response of the structure with k = 5, M = 1.7088, the structure unstable. Compared to the case of the structure that the same size but k = 1 as shown in Figure 2, we find the structure unstable with M = 2.7669.

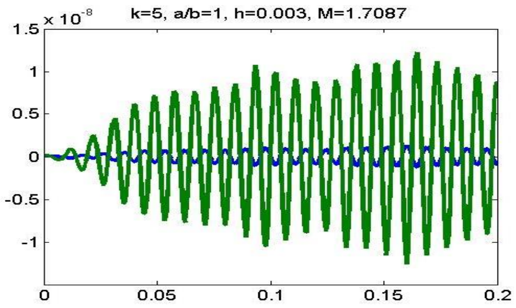

__Figure 7.__ The structure is stable with M = 1.7087

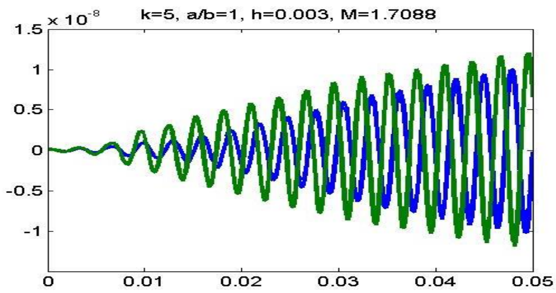

__Figure 8.__ The structure is unstable with M = 1.7088

The thickness of the structure has a great influence on stability. With the thickness h = 0.0045, M = 5.5393, the structure is stable, the graph shown in Figure 9 is the harmonic function, Figure 10 shows the instability of the structure with h = 0.0040, M = 5.5393.

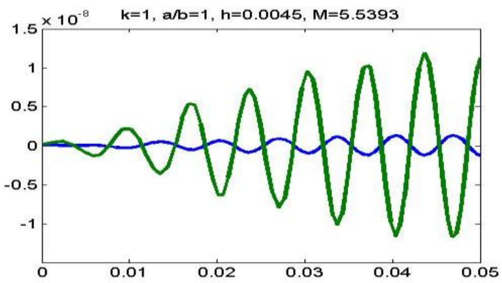

Figure 9. *The structure is stable with h = 0.0045*

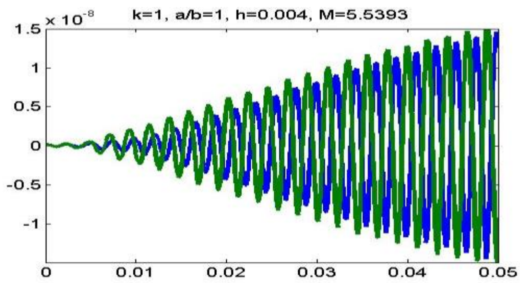

Figure 10. *The structure is unstable with h = 0.0040*

III. CONCLUSION

There are some research results on dynamic stability behavior of cylindrical panel structures that are in aerodynamic flux with supersonics speed. However, in the previous results, other authors have mainly used the finite element method and they have only considered the individual cases associated with a specific oscillation mode. In this paper, we considered the general case with any oscillation mode and instead of the previous common method, we used the analytical method in combination with the numerical method to investigate.

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