STUDYING THE STABILITY OF NONLINEAR DYNAMIC SYSTEM USING AERODYNAMIC THEORY

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Abstract: *The dynamic analysis of the toroidal shell segment has been studied and published by a number of authors, but the former authors mainly used finite element method and only considered specific cases. In this paper, the author used mathematical analytical method combined with numerical method to solve problem and considered with the general case; bringing the partial derivative differential equations system that describe the motion to the ordinary differential equation system, which depends on the geometric parameters of the structure. Compared to finite element method, mathematical analytical method is more general.*

Keywords. *Stability, nonlinear vibration, toroidal shell, aerodynamic.*

I. INTRODUCTION

Toroidal shell segments are widely used as joint components or accessories in different engineering fields for their specific advantages. They have many important applications in the fields of physics and engineering, such as, nuclear power, satellite support structures, rocket fuel tanks, fusion reactor vessels, ocean engineering,… A lot of scientists have been studying this issue. Jordan [1] and Panagiotopoulos [2] had researched the stability of the toroidal shells by finite difference method (FDM). Galletly and Blachut [3, 4] had studied the stability of closed toroidal shells with circularor noncircular crosssections. Blachut and Jaiswal [6], Blachut and Smith [8] have studied the stability of toroidal shells as segment or entirety by finite element method FEM and model tests,… and many other authors [5, 7, 10].

Recently, researchers have emphasized the importance of toroidal shell applications and many scientific articles have been published. The experimental and numerical results both reveal that the initial geometric imperfection would mainly determine the model's final failure mode, critical pressure and collapse deformation. Further parametric study is made to study the buckling property of toroidal shell, including failure mode and critical pressure loading by varying the structural parameters. This investigation will lay a good foundation for deriving a theoretical solution for the buckling of toroidal shell in the near future [see 9, 11, 12].

The load of aerodynamics is considered when calculating the structure of the flying equipment and the structure has high height or length, as skyscraper, cable-stayed bridge, antenna tower. The aerodynamic stability of the technical systems is one of top criteria in calculating the structure of structures subjected to aerodynamic load. The stability of toroidal shell segments structures has been studied by many scientists. However, in their publications concerning the stability of these structural forms, the authors have mainly used the finite element method and the finite difference method, moreover, they have considered with only special cases [3, 5, 6, 7, 8].

In the main content of the presentation below, we study the dynamic stability behavior of the toroidal shell segments structure that are in the aerodynamic flux that moves supersonic, establishes dynamics governing equations and investigates nonlinear vibration of the structure. In the previous results, other authors only considered the individual cases associated with a specific oscillation mode. In this paper, we considered the general case with any oscillation mode. On qualitatively, by analytic method, it was explicitly established the 2^{nd} order nonlinear differential equations system which depends on geometric parameters as well as depends on the oscillator mode. By the numerical method applied to the established model, can quantitative research on the stability of the technical system.

II. CONTENT

2.1. The nonlinear vibration of the structure

Consider a toroidal shell segment that material composition of the shell varies smoothly along the thickness of the structure, inner surface is ceramic-rich and the outer surface is metal-rich. The shell of thickness h, length a, the radius of curvature R. Symbol $k \geq 0$ is the volume - fraction index, z is the thickness coordinate of the shell and $z \in \left[-\frac{h}{2}, \frac{h}{2}\right]$, $E(z)$ is Young modulus, $\rho(z)$ is mass density $\rho(z)$ of the material; E_m, E_c and $\rho_m(z), \rho_c(z)$ are respectively the Young modulus and mass densities of the metal and of the ceramic.

For the middle surface of a toroidal shell segment, we have: $r = L - R(1 - sin\varphi)$, where L is the equator radius and φ is the angle between the axis of revolution and the normal to the shell surface. For a sufficiently shallow toroidal shell in the region of the equator of the torus, the angle φ is approximately equal to $\frac{\pi}{2}$, thus $sin\varphi = 1, cos\varphi = 0$ and $r = L.$ Suppose the Cartesian coordinates (x_1, x_2, z) include x_1, x_2 *and* z axes are in the meridian, parallel and inward radial directions, respectively.

With ε_1^0 and ε_2^0 are normal strains, γ_{12}^0 is the shear strain at the middle surface of the toroidal shell and χ_{ij} are the curvatures, according to the classical shell theory (see [12]), the strains at the middle surface and curvatures are related to the displacement components u, v, w in the x_1, x_2, z coordinate directions are defined by:

$$\varepsilon_1^0 = \frac{\partial u}{\partial x_1} - \frac{\mathrm{w}}{R} + \frac{1}{2}\left(\frac{\partial \mathrm{w}}{\partial x_1}\right)^2, \ \varepsilon_2^0 = \frac{\partial v}{\partial x_2} - \frac{\mathrm{w}}{L} + \frac{1}{2}\left(\frac{\partial \mathrm{w}}{\partial x_2}\right)^2,$$

$$\gamma_{12}^0 = \frac{\partial u}{\partial x_2} + \frac{\partial v}{\partial x_1} + \frac{\partial w}{\partial x_1}\frac{\partial w}{\partial x_2}, \ \chi_1 = \frac{\partial^2 w}{\partial x_1^2}, \ \chi_2 = \frac{\partial^2 w}{\partial x_2^2}, \ \chi_{12} = \frac{\partial^2 w}{\partial x_1 \partial x_2},$$

The strains across the toroidal shell thickness at a distance z from the middle surface determined as follows:

$$\varepsilon_1 = \varepsilon_1^0 - z\chi_1, \ \varepsilon_2 = \varepsilon_2^0 - z\chi_2, \ \gamma_{12} = \gamma_{12}^0 - 2z\chi_{12}. \tag{1}$$

The deformation compatibility equation is:

$$\frac{\partial^2 \varepsilon_1^0}{\partial x_2^2} + \frac{\partial^2 \varepsilon_2^0}{\partial x_1^2} - \frac{\partial^2 \gamma_{12}^0}{\partial x_1 \partial x_2} = \left(\frac{\partial^2 w}{\partial x_1 \partial x_2} \right)^2 - \frac{\partial^2 w}{\partial x_1^2}\frac{\partial^2 w}{\partial x_2^2} - \frac{1}{R}\frac{\partial^2 w}{\partial x_2^2} - \frac{1}{L}\frac{\partial^2 w}{\partial x_1^2} \tag{2}$$

The relationship between stress and deformation of the shell material is described by the Hooke's law. The forces and moments resultants of the shell are determined:

$$\begin{cases} N_1 = \int_{-\frac{h}{2}}^{\frac{h}{2}} \frac{E(z)}{1-\nu^2}(\varepsilon_1 + \nu\varepsilon_2)\,dz = \frac{E_1}{1-\nu^2}(\varepsilon_1^0 + \nu\varepsilon_2^0) - \frac{E_2}{1-\nu^2}(\chi_1 + \nu\chi_2) \\ N_2 = \int_{-\frac{h}{2}}^{\frac{h}{2}} \frac{E(z)}{1-\nu^2}(\varepsilon_2 + \nu\varepsilon_1)\,dz = \frac{E_1}{1-\nu^2}(\varepsilon_2^0 + \nu\varepsilon_1^0) - \frac{E_2}{1-\nu^2}(\chi_2 + \nu\chi_1) \\ N_{12} = \int_{-\frac{h}{2}}^{\frac{h}{2}} \frac{E(z)}{2(1+\nu)}\gamma_{12}\,dz = \frac{E_1}{2(1+\nu)}\gamma_{12}^0 - \frac{E_2}{1+\nu}\chi_{12}, \end{cases} \tag{3}$$

In (3), we denote:

$$\begin{cases} E_1 = \int_{-\frac{h}{2}}^{\frac{h}{2}} E(z)\,dz = \left(E_m + \frac{E_c - E_m}{k+1} \right) h, \\ E_2 = \int_{-\frac{h}{2}}^{\frac{h}{2}} E(z)\,z\,dz = \frac{E_c - E_m}{2(k+1)(k+2)} kh^2, \\ E_3 = \int_{-\frac{h}{2}}^{\frac{h}{2}} E(z)\,z^2\,dz = \left\{ \frac{E_m}{12} + \left[\frac{1}{k+3} - \frac{1}{k+2} + \frac{1}{4(k+1)} \right] (E_c - E_m) \right\} h^3. \end{cases}$$

The strain-force resultant relations are determined from equation (3)

$$\begin{cases} \varepsilon_1^0 = \dfrac{1}{E_1}\left(N_1 - \nu N_2\right) + \dfrac{E_2}{E_1}\chi_1, \\ \varepsilon_2^0 = \dfrac{1}{E_1}\left(N_2 - \nu N_1\right) + \dfrac{E_2}{E_1}\chi_1, \\ \gamma_{12}^0 = 2\left(\dfrac{1+\nu}{E_1}N_{12} + \dfrac{E_2}{E_1}\chi_{12}\right). \end{cases} \tag{4}$$

From (4), we have:

$$\begin{cases} M_1 = \dfrac{E_2}{E_1}N_1 - \dfrac{E_1E_3 - E_2^2}{E_1\left(1-\nu^2\right)}\left(\chi_1 + \nu\chi_2\right), \\ M_2 = \dfrac{E_2}{E_1}N_2 - \dfrac{E_1E_3 - E_2^2}{E_1\left(1-\nu^2\right)}\left(\chi_2 + \nu\chi_1\right), \\ M_{12} = \dfrac{E_2}{E_1}N_{12} - \dfrac{E_1E_3 - E_2^2}{E_1\left(1-\nu^2\right)}\chi_{12}, \end{cases} \tag{5}$$

According to the nonlinear Piston theory, when the direction of the gas flow along the x_1 axis, the intensity of aerodynamic forces that affect the structure is defined by (see [13]).

$$q_0 = -\gamma P_\infty\left(\frac{U}{a_\infty}\frac{\partial w}{\partial x_1} + \frac{1}{a_\infty}\frac{\partial w}{\partial t}\right) - \frac{\gamma\left(\gamma+1\right)}{4}P_\infty\left(\frac{U}{a_\infty}\frac{\partial w}{\partial x_1} + \frac{1}{a_\infty}\frac{\partial w}{\partial t}\right)^2 \tag{6}$$

Where γ is the heat capacity of gas, a_∞ is the speed of sound, P_∞ is the gas pressure without perturbation, U is the speed of the gas flow.

The stress function φ is determined as follows:

$$N_1 = \frac{\partial^2\varphi}{\partial x_2^{\,2}}\ ,\ N_2 = \frac{\partial^2\varphi}{\partial x_1^{\,2}}\ ,\ N_{12} = -\frac{\partial^2\varphi}{\partial x_1\,\partial x_2},\ \rho_0 = \left(\rho_m + \frac{\rho_c - \rho_m}{k+1}\right)h, \tag{7}$$

then, the nonlinear equilibrium equations of a toroidal shell segment under a lateral pressure q_o, by the classical shell theory have form:

$$\frac{\partial^2 M_1}{\partial x_1^{\,2}} + 2\frac{\partial^2 M_{12}}{\partial x_1\,\partial x_2} + \frac{\partial^2 M_2}{\partial x_2^{\,2}} + N_1\frac{\partial^2 w}{\partial x_1^{\,2}} + 2N_{12}\frac{\partial^2 w}{\partial x_1\,\partial x_2} + N_2\frac{\partial^2 w}{\partial x_2^{\,2}} + \frac{N_1}{R} + \frac{N_2}{L} + q_0 = \rho_0\frac{\partial^2 w}{\partial t^2}. \tag{8}$$

From (2), (4) and (7), we transform the deformation compatibility equation into:

$$\frac{1}{E_1}\Delta\Delta\varphi = -\frac{1}{R}\frac{\partial^2 w}{\partial x_2^{\,2}} - \frac{1}{L}\frac{\partial^2 w}{\partial x_1^2} + \left(\frac{\partial^2 w}{\partial x_1\,\partial x_2}\right)^2 - \frac{\partial^2 w}{\partial x_1^{\,2}}\frac{\partial^2 w}{\partial x_2^{\,2}}. \tag{9}$$

From (5), (6) and (7), the equation (8) can be rewritten as follows:

$$\rho_0 \frac{\partial^2 w}{\partial t^2} + \gamma P_\infty \left(\frac{U}{a_\infty} \frac{\partial w}{\partial x_1} + \frac{1}{a_\infty} \frac{\partial w}{\partial t} \right) + \frac{\gamma(\gamma+1)P_\infty}{4} \left(\frac{U}{a_\infty} \frac{\partial w}{\partial x_1} + \frac{1}{a_\infty} \frac{\partial w}{\partial t} \right)^2 + \frac{E_1 E_3 - E_2^2}{E_1(1-\nu^2)} \Delta\Delta w$$
$$+ 2 \frac{\partial^2 \varphi}{\partial x_1 \partial x_2} \frac{\partial^2 w}{\partial x_1 \partial x_2} - \frac{\partial^2 \varphi}{\partial x_2{}^2} \frac{\partial^2 w}{\partial x_1{}^2} - \frac{\partial^2 \varphi}{\partial x_1{}^2} \frac{\partial^2 w}{\partial x_2{}^2} - \frac{1}{R} \frac{\partial^2 \varphi}{\partial x_2{}^2} - \frac{1}{L} \frac{\partial^2 \varphi}{\partial x_1^2} = 0. \tag{10}$$

The system of nonlinear equations (9) and (10) has functions that need to be determined φ and w, describing the nonlinear vibration of the toroidal shell segment structure under aerodynamic force. This system is used to investigate the dynamic stability of the structure.

2.2. The system of motion differential equations

With $f_1(t)$, $f_2(t)$ as the time-dependent total amplitudes, $n, m \in \mathbb{Z}^+$, we will find solution of the system (9) - (10) in the form:

$$w = \left[f_1(t) \sin \frac{m\pi x_1}{a} + f_2(t) \sin \frac{(m+1)\pi x_1}{a} \right] \sin \frac{n x_2}{2L}, \tag{11}$$

this solution satisfies the simply supported boundary conditions as follows

$$\left(w = M_1 = N_2 = N_{12} \right)\Big|_{(x_1 = 0,\, x_1 = a)} = 0 \tag{12}$$

and satisfies the initial condition

$$\begin{cases} f_1(0) = f_1^{(0)}, \ \dot{f}_1(0) = f_{11}^{(0)} \\ f_2(0) = f_2^{(0)}, \ \dot{f}_2(0) = f_{21}^{(0)}. \end{cases} \tag{13}$$

From (11), solve the system of equations (9) - (10) and by setting

$$\begin{cases} \varphi_1 = \dfrac{E_1 f_1 a^2}{\pi^2 \left(m^2 + n^2\lambda^2\right)^2} \left(\dfrac{m^2}{L} + \dfrac{n^2\lambda^2}{R} \right), \ \varphi_2 = \dfrac{E_1 f_2 a^2}{\pi^2 \left[(m+1)^2 + n^2\lambda^2\right]^2} \left[\dfrac{(m+1)^2}{L} + \dfrac{n^2\lambda^2}{R} \right], \\ \varphi_3 = \dfrac{E_1 \left[f_1^2 m^2 + f_2^2 (m+1)^2 \right]}{32 n^2 \lambda^2}, \ \varphi_4 = \dfrac{E_1 f_1^2 n^2 \lambda^2}{32 m^2}, \varphi_5 = \dfrac{E_1 f_2^2 n^2 \lambda^2}{32 (m+1)^2}, \ \varphi_6 = \dfrac{E_1 f_1 f_2 n^2 \lambda^2}{4(2m+1)^2} \\ \varphi_7 = \dfrac{-E_1 f_1 f_2 n^2 \lambda^2}{4}, \varphi_8 = \dfrac{-E_1 f_1 f_2 n^2 \lambda^2}{4\left[(2m+1)^2 + 4n^2\lambda^2\right]^2}, \ \varphi_9 = \dfrac{E_1 f_1 f_2 n^2 (2m+1)^2 \lambda^2}{4\left(1 + 4n^2\lambda^2\right)^2}. \end{cases} \tag{14}$$

we obtain:

$$\varphi = \varphi_1 \sin \frac{m\pi x_1}{a} \sin \frac{n x_2}{2L} + \varphi_2 \sin \frac{(m+1)\pi x_1}{a} \sin \frac{n x_2}{2L}$$
$$+ \varphi_3 \cos \frac{n x_2}{L} + \varphi_4 \cos \frac{2m\pi x_1}{a} + \varphi_5 \cos \frac{2(m+1)\pi x_1}{a} + \varphi_6 \cos \frac{(2m+1)\pi x_1}{a} \tag{15}$$
$$+ \varphi_7 \cos \frac{\pi x_1}{a} + \varphi_8 \cos \frac{(2m+1)\pi x_1}{a} \cos \frac{n x_2}{L} + \varphi_9 \cos \frac{\pi x_1}{a} \cos \frac{n x_2}{L}.$$

Using the Galerkin method for the system of nonlinear differential equations (9) - (10), we obtained the system of differential equations of motion (16) - (17) below, which is considered with the case where $m \in ¢^{+}$ is an even number:

$$\rho_0 \frac{a^4}{\pi^4}\ddot{f}_1 + \frac{E_1E_3 - E_2^2}{E_1(1-\nu^2)}\left(m^2+n^2\lambda^2\right)^2 f_1 + \frac{a^4E_1\, f_1}{\pi^4\left(m^2+n^2\lambda^2\right)^2}\left(\frac{m^2}{L}+\frac{n^2\lambda^2}{R}\right)^2$$

$$+\frac{\gamma P_\infty a^4}{\pi^4}\left[-f_2\frac{4Um(m+1)}{a_\infty a(2m+1)}+\frac{\dot{f}_1}{a_\infty}\right]+\frac{f_1 f_2^2 n^4\lambda^4 E_1}{16}\times$$

$$\times\left\{4+\frac{m^2(m+1)^2}{n^4\lambda^4}+\frac{(2m+1)^4}{(1+4n^2\lambda^2)^2}+\frac{1}{[(2m+1)^2+4n^2\lambda^2]^2}\right\}+\frac{f_1^3E_1}{16}\left(m^4+n^4\lambda^4\right)$$

$$+\frac{2 f_1 f_2\, a^2 n\,\lambda^2\delta_n\, E_1}{\pi^4(3m+1)\left(m^2-1\right)L}\left\{\frac{\left[4n^2\lambda^2L+R\right](3m+1)}{3R\left(1+4n^2\lambda^2\right)^2}+\frac{m(m-1)\left[4n^2\lambda^2L+(2m+1)^2R\right]}{3R\left[(2m+1)^2+4n^2\lambda^2\right]^2}+4m^2\right\}$$

$$+\frac{16 f_1 f_2\, a^2 n\, m^2\lambda^2\delta_n\, E_1}{3\pi^4(3m+1)\left(m^2-1\right)RL}\left\{\frac{\left(m^2R+n^2\lambda^2L\right)}{\left(m^2+n^2\lambda^2\right)^2}+\frac{\left[(m+1)^2R+n^2\lambda^2L\right]}{\left[(m+1)^2+n^2\lambda^2\right]^2}\right\}\left[2m^2+(m+1)^2\right]$$

$$-\frac{8P_\infty\gamma(\gamma+1)a^2\delta_n}{3n\pi^4(3m+1)}\left[\frac{U^2 f_1 f_2 m^2(m+1)}{a_\infty{}^2(m-1)}+\frac{2\dot{f}_1\dot{f}_2 m^2}{\left(m^2-1\right)a_\infty^2}\right]=0, \qquad (16)$$

$$\rho_0\frac{a^4}{\pi^4}\ddot{f}_2+\frac{E_1E_3-E_2^2}{E_1(1-\nu^2)}[(m+1)^2+n^2\lambda^2]^2 f_2+\frac{\left[(m+1)^2R+n^2\lambda^2L\right]^2 a^4E_1\, f_2}{R^2L^2\pi^4\left[(m+1)^2+n^2\lambda^2\right]^2}$$

$$+\frac{\gamma P_\infty a^4}{\pi^4}\left[f_1\frac{4Um(m+1)}{a_\infty a(2m+1)}+\frac{\dot{f}_2}{a_\infty}\right]+\frac{f_2 f_1^2 n^4\lambda^4E_1}{16}\times$$

$$\times\left\{4+\frac{m^2(m+1)^2}{n^4\lambda^4}+\frac{(2m+1)^4}{(1+4n^2\lambda^2)^2}+\frac{1}{[(2m+1)^2+4n^2\lambda^2]^2}\right\}+\frac{f_2^3E_1}{16}[(m+1)^4+n^4\lambda^4]-$$

$$+\frac{2 f_1^2a^2n\,\lambda^2\delta_n\, E_1}{\pi^4(3m+1)\left(m^2-1\right)}\left[\frac{8\left(3m^2+2m+1\right)\left(m^2R+n^2\lambda^2L\right)}{3RL\left(m^2+n^2\lambda^2\right)^2}+\frac{(m+1)^2}{2L}+\frac{m^2(3m+1)(m-1)}{3Rn^2\lambda^2}\right]$$

$$+\frac{2 f_2{}^2a^2n\,\lambda^2\delta_n E_1}{3\,\pi^4RL}\left\{\frac{8(m+1)}{\left[(m+1)^2+n^2\lambda^2\right]^2}+\frac{1}{(m+1)n^2\lambda^2}\right\}\left[(m+1)^2R+n^2\lambda^2L\right]-$$

$$-\frac{4\gamma(\gamma+1)P_\infty a^2\delta_n}{3n\pi^4}\left\{\frac{U^2}{a_\infty{}^2}\left[\frac{\left(m^2-2m-1\right)f_1^2}{\left(m^2-1\right)(3m+1)}+\frac{(m+1)f_2^2}{3}\right]+\frac{2\dot{f}_1^2m^2}{(3m+1)\left(m^2-1\right)a_\infty^2}\right\}=0 \quad (17)$$

And the differential equations system (18) - (19) below, which is considered with case $m \in ¢^{+}$ is an odd number:

$$\rho_0 \frac{a^4}{\pi^4}\ddot{f}_1 + \frac{E_1E_3 - E_2^2}{E_1(1-\nu^2)}\left(m^2+n^2\lambda^2\right)^2 f_1 + \frac{\left(m^2R+n^2\lambda^2L\right)^2 a^4E_1 f_1}{R^2L^2\pi^4\left(m^2+n^2\lambda^2\right)^2} + \frac{\gamma P_\infty a^4}{\pi^4}\times$$

$$\times\left[-f_2\frac{4Um(m+1)}{a_\infty a(2m+1)} + \frac{\dot{f}_1}{a_\infty}\right] + \frac{f_1^3E_1}{16}\left(m^4+n^4\lambda^4\right) + \frac{f_1f_2^2n^4\lambda^4E_1}{16}\times$$

$$\left\{4+\frac{m^2(m+1)^2}{n^4\lambda^4}+\frac{(2m+1)^4}{(1+4n^2\lambda^2)^2}+\frac{1}{[(2m+1)^2+4n^2\lambda^2]^2}\right\} + \frac{2f_1^2a^2n\lambda^2\delta_n E_1}{3\pi^4}\times$$

$$\times\left[\frac{8m^2\left(n^2\lambda^2L+m^2R\right)}{RL\left(m^2+n^2\lambda^2\right)^2}+\frac{1}{2mL}+\frac{m}{2Rn^2\lambda^2}\right] + \frac{2f_2^2a^2n\lambda^2\delta_n E_1}{\pi^4(3m+2)(m+2)}\times$$

$$\left\{\frac{16(m+1)^2\left(3m^2+4m+2\right)\left[n^2\lambda^2L+(m+1)^2R\right]}{3mRL\left[(m+1)^2+n^2\lambda^2\right]^2}+\frac{m}{2L}+\frac{(m+1)^2(m+2)(3m+2)}{2Rmn^2\lambda^2}\right\}-$$

$$-\left\{\frac{U^2}{a_\infty^2}\left[\frac{f_1^2m}{3}+\frac{f_2^2(m+1)^2\left(m^2+4m+2\right)}{m(m+2)(3m+2)}\right]+\frac{2}{a_\infty^2m}\left[\frac{\dot{f}_1^2}{3}+\frac{\dot{f}_2^2(m+1)^2}{(m+2)(3m+2)}\right]\right\}\times$$

$$\times\frac{4\gamma(\gamma+1)P_\infty a^2\delta_n}{3n\pi^4}=0, \qquad (18)$$

$$\rho_0\frac{a^4}{\pi^4}\ddot{f}_2+\frac{E_1E_3-E_2^2}{E_1(1-\nu^2)}[(m+1)^2+n^2\lambda^2]^2 f_2+\frac{\left[(m+1)^2R+n^2\lambda^2L\right]^2a^4E_1 f_2}{R^2L^2\pi^4\left[(m+1)^2+n^2\lambda^2\right]^2}$$

$$+\frac{\gamma P_\infty a^4}{\pi^4}\left[f_1\frac{4Um(m+1)}{a_\infty a(2m+1)}+\frac{\dot{f}_2}{a_\infty}\right]+\frac{f_2f_1^2n^4\lambda^4E_1}{16}\times\left\{4+\frac{m^2(m+1)^2}{n^4\lambda^4}+\frac{(2m+1)^4}{(1+4n^2\lambda^2)^2}+\right.$$

$$\left.+\frac{1}{[(2m+1)^2+4n^2\lambda^2]^2}\right\}+\frac{f_2^3E_1}{16}[(m+1)^4+n^4\lambda^4]-\frac{2f_1f_2a^2n\lambda^2\delta_n E_1(m+1)}{3\pi^4mL(3m+2)(m+2)}\times$$

$$\left\{\frac{(2m+1)^2\left(4n^2\lambda^2+R\right)(3m+2)}{\left(1+4n^2\lambda^2\right)^2R}-\frac{\left[(2m+1)^2R+4n^2\lambda^2L\right](m+2)}{R\left[(2m+1)^2+4n^2\lambda^2\right]^2}+12\right\}+$$

$$\frac{16f_1f_2a^2n\lambda^2\delta_n E_1(m+1)}{3m\pi^4(3m+2)(m+2)RL}\left\{\frac{\left(m^2R+n^2\lambda^2L\right)}{\left(m^2+n^2\lambda^2\right)^2}+\frac{\left[(m+1)^2R+n^2\lambda^2L\right]}{\left[(m+1)^2+n^2\lambda^2\right]^2}\right\}\left[2(m+1)^2+m^2\right]$$

$$-\frac{8\gamma(\gamma+1)P_\infty a^2\delta_n}{3n\pi^4}\left\{\frac{U^2}{a_\infty^2}\frac{f_1f_2m(m+1)^2}{(3m+2)(m+2)}+\frac{1}{a_\infty^2}\left[\frac{\dot{f}_2^2}{3(m+1)}+\frac{2\dot{f}_1\dot{f}_2(m+1)^2}{m(3m+1)(m+2)}\right]\right\}=0. \qquad (19)$$

These systems of equations above are established by analytical method, describes flutter and the flutter frequency of the toroidal shell segment, this system is used to investigate the nonlinear flutter response of structure in the moving supersonic airflow. By numerical method applied to these systems of equations, using RK4 method in Matlab, there can be explicit estimates of the stability of the structure depending on the its geometric parameters.

III. CONCLUSION

In this paper, the author studied the stability of the toroidal shell segments structures while they are affected by aerodynamics. Based on the Donnell shell theory, taking into account the strains components of the geometrical nonlinearity in Von Kármán sense, by analytical method, the author has analyzed the effect of aerodynamic on the structural and has established a basic mathematical model which description the motion of toroidal shell structures, simultaneously, using this model to study the stability behavior of the structures in aerodynamic flux with a supersonics speed.

References

[1]. Jordan, P.F., 1973. Buckling of toroidal shells under hydrostatic pressure. *J. AIAA 11, 1439-1441.*

[2]. Panagiotopoulos, G.D., 1985. Stress and stability analysis of toroidal shells. *20. I. J. Press. Vessel. Pip., pp. 47-100 .*

[3]. Galletly, G.D., Blachut, J., 1995. Stability of complete circular and non-circular toroidal shells. *Proc. Inst. Mech. Eng. 209, 245-255.*

[4]. Galletly, G.D., 1998. Elastic buckling of imperfect circular toroidal shells under external pressure. *J. Process Mech. Eng. 212, 197-209.*

[5]. Parnell TK. Numerical improvement of asymptotic solution for shells of revolution with application to toroidal shell segments. *Comput struct 1983:16:109-117.*

[6]. Blachut, J., Jaiswal, O.R., 2000. On buckling of toroidal shells under external pressure. *Comput. Struct. 77, 233-251.*

[7]. Blachut, J., 2003. Collapse tests on externally pressurized toroids. *J. Press. Vessel. Technol. 125, 91-96.*

[8]. Blachut, J., Smith, P., 2008. Buckling of multi-segment underwater pressure hull. *Ocean Eng. 35, 247-260.*

[9]. Blachut, J., 2014. Experimental perspective on the buckling of pressure vessel components. *Appl. Mech. Rev. 66, 011003, 1-24.*

[10]. Zhan, H.J., Redekop, D., 2008. Vibration, buckling and collapse of ovaloid toroidal tanks. *Thin-Walled Struct. 46, 380-389.*

[11]. Qinghai Du, Weicheng Cui, Bowen Zhang. Buckling characteristics of a circular toroidal shell with stiffened ribs. *Ocean Engineering 108 (2015) 325-335.*

[12]. Bich DH, Ninh DG, Kien BH, David H. Nonlinear dynamic analyses of eccentrically stiffened functionally graded toroidal shell segments surrounded by elastic foundation in thermal environment. *Compos. Part B 2016, 355-373.*

[13]. Chen X., Kareem A., Haan Jr. F. L, 2000. Nonlinear Aerodynamic Analysis of Bridges under Turbulent Winds: The New Frontier in Bridge Aerodynamic. *University of Notre Dame, IN 46556-0767, USA.*