REFINEMENT TO CALCULATION OF STABILIZING MOMENTS OF STEERABLE WHEELS

MIKHAIL P. MALINOVSKY
MADI, 64, Leningradsky prosp., Moscow, 125319, Russia
Corresponding author's email: ntbmadi@gmail.com

Abstract: *One of the most difficult tasks in designing a wheeled vehicle is the kinematic matching of the steering and suspension. In addition to providing the required cross-country capability, this task involves choosing the optimum alignment angles of steerable wheels and calculating the reactive force on the steering wheel that depends on the magnitude of stabilizing moments and turning resistance forces. The author of the article carried out a general analysis of the effect of the alignment angles on some operational properties, and also proposed a mathematical model for calculating the moment of stabilization.*

Keywords: *wheeled vehicles; steering; stabilization of steerable wheels; toe; camber; Caster angle; kingpin inclination.*

I. INTRODUCTION

The following parameters and properties must be taken into account when jointly designing the steering and suspension of a wheeled machine: steerability [1], maneuverability [2], driving stability [3], passability [4], informativeness [5], and steering sensitivity [6], the maximum allowable force on the steering wheel and stabilizing moments on steerable wheels [7]. In order to achieve an optimal ratio between the parameters and properties listed above, the steerable wheels are initially set at specific angles, among which are camber angle β, toe angle γ, Caster angle ζ, and kingpin lateral inclination ξ. The concept of "included angle" χ is also used, which depends on the axle geometry and determines its position relative to the axis of rotation of the wheel. The angle χ is a constructive constant and is found as the sum of the camber angle β and the kingpin lateral inclination ξ. As β decreases, ξ increases and vice versa.

For a wheeled vehicle, the position of the wheels is considered ideal when their rolling surfaces are perpendicular to the road surface, parallel to the plane of symmetry of the body and the trajectory of movement. In this case, the grip of the wheels with the road as much as possible, and the power loss due to friction and tire wear is minimal. Installation angles are regulated in a static state under the action of weight G. However, in motion the picture changes. Under acceleration, body trim arises. When turning on a wheel drive, the centrifugal force F_j acts, as a result of which the body acquires roll, and lateral forces F_Y and counteracting lateral reactions R_Y occur on the steerable wheels.

The author proposed a refined method of calculating the moments of stabilization of the steerable wheels.

II. EFFECT OF INSTALLATION ANGLES ON MOTION PARAMETERS

The angles of installation of the steerable wheels are interrelated: each of those compensates for the shortcomings caused by the presence of other angles.

With an increase of the kingpin lateral inclination ξ and the positive camber $\beta > 0$, the distance from the center of the contact patch to the point of intersection of the axis of rotation with the road surface - the so-called "scrub radius" r_f – decreases. Proportionally to r_f, the effect of weight stabilization increases, the moment of resistance to turning in place M_f, the forces loading the steering linkage parts during movement, as well as the informativeness of the steering when the longitudinal reactions imbalance R_X from irregularities or non-uniformity of braking forces.

With a negative scrub radius ($r_f < 0$), the axis of rotation lies outside the wheel rolling plane. In this case, a change in the longitudinal reaction of the R_X (for example, due to an imbalance of braking forces or a sharp increase in the rolling resistance force at a tire puncture) creates an unfolding moment around the center of mass of the wheeled vehicle, however, the steerable wheels turn in the opposite direction, which contributes to the stability of straight-line motion.

Positive camber $\beta > 0$ ensures that the wheel is upright regardless of the kinematics and compliance of the suspension elements, gaps in the hub bearings and bushings of the pivots, which leads to improved tire adhesion and reduced one-sided tread wear, and when straight-ahead driving on a level road, unloads the bearing, which increases its durability [8]. However, when maneuvering and traveling irregularities, dynamic loads level this effect.

When rolling inclined wheel under the action of its own gravity tends to turn in the direction of inclination. As a result of the elastic deformation of the tire, an additional lateral force arises in the contact patch, acting in the same direction. The negative camber $\beta<0$ has a positive effect on curvilinear motion stability and controllability. The lateral reaction of R_Y, which counteracts the centrifugal forces and contributes to keeping the wheel vehicle on a curved path, reaches a maximum at small negative camber angles. Negative camber leads to undesirable consequences – causes increased wear of the inner tread zone, reduces the stability of the rectilinear movement and the efficiency of the longitudinal accelerations during the rectilinear movement due to the reduction of the contact patch area.

With increasing weight, the wheels tend to decrease β. The heel and trim of the body, which occur respectively during lateral and longitudinal accelerations, are accompanied by the redistribution of R_Z and the displacement of suspension elements, leading to a change in β.

A positive toe $\gamma > 0$ compensates for the influence of the longitudinal forces and camber thrust acting on the wheel in straight-line motion, thereby reducing sawtooth tire wear and fuel consumption. The actual toe is determined by the elastic deformation of the steering and suspension elements, the compensation of gaps in hinges and wheel bearings and depends on the drive. With rear wheel drive while driving, as well as with braking, the force of inertia pushes the pivots forward, and the wheels turn around them under the force of rolling resistance in the direction of toe-out ($\Delta\gamma < 0$). With front-wheel drive, on the contrary, the thrust force creates a turning moment regarding the pivots, and the wheels tend to increase positive toe ($\Delta\gamma > 0$), therefore there are cases of neutral adjustment of toe ($\gamma = 0$) among front-wheel drive vehicles. With all-wheel drive, $\Delta\gamma$ depends on the distribution of torque along the axes of the wheeled vehicle and is inversely proportional to the longitudinal response of the R_X. The ideal solution is variable toe.

A positive toe $\gamma > 0$ also increases the stability of a rectilinear motion when it hits an unevenness. The increased resistance force turns the wheel in the direction of decreasing toe. Through the steering drive, the effect is transmitted to the second wheel, the toe of which, on the contrary, increases, which counteracts the disturbance.

When $\gamma < 0$, a destabilizing moment appears, contributing to yaw in straight-line motion, but at the same time steering sensitivity increases at the entrance to the turn.

The positive Caster angle $\zeta > 0$ (when the axis of rotation of the wheel crosses the road surface in front of the center of the contact patch) provides constructive speed stabilization during curvilinear movement, increases the stability of movement under the action of random oscillatory disturbances (due to unevenness of the road, wheel imbalance) or sudden lateral force (due to gust of wind, when moving across the slope), provides steering informativeness due to the reactive forces transmitted to the steering wheel. In addition, ζ affects the weight stabilization: the top of the arc, which is described by the trunnion, is displaced so that the trunnions of both wheels in a neutral position are on the descending part of the arc. Thus, when turning, one of them moves in an arc upwards, the other – downwards. As a result, the loading of one of the wheels and its weight stabilization increase. This effect is also used to optimize the roll of the front part of the body of a wheeled vehicle when turning.

The redistribution of weight and body trim significantly affect the Caster angle. With an excessive increase in ζ, it is difficult to enter the car into a turn, the load on the amplifier and steering gear parts increases, as well as the force on the steering wheel, which, when exiting the turn, abruptly returns to the neutral position. With a negative Caster angle ($\zeta < 0$), the force on the steering wheel decreases.

As is known, by nature, stabilization of the steering wheels is of two types – speed stabilization and weight stabilization. We shall consider those separately on the example of a three-axle truck «KamAZ-5320» with tires 260R508, which has the following technical characteristics:

- Camber angle $\beta = 1°$;

- Toe angle $\gamma = 20'$;

- Caster angle $\zeta = 2°30'$;

- Longitudinal displacement of the pivot relative to the wheel center $e = 0$;

- Kingpin lateral inclination $\xi = 8°$;

- Front axle load $m_1 = 4375$ kg;

- Mass center height $h_C = 1{,}278$ m;

- Front track $B_1 = 2{,}026$ m;

- Wheelbase L = 3,85 m;

- Stub axle length $b_1 = 0{,}13$ m;

- Free wheel radius $r_w = 0{,}509$ m;

- Static wheel radius $r_{st} = 0{,}475$ m (at nominal pressure $p_{Tnom} = 0{,}64$ MPa and rated load m_{Tnom} = 2240 kg).

Traditional methods of calculating the stabilizing moment do not take into account the redistribution of weight and the dependence of the wheel radius on the vertical load. The angles of rotation of the inner and outer steered wheels are not equal ($\alpha_i \neq \alpha_e$), which means that the stabilizing moments on the right and left wheels are different.

III. SPEED STABILIZATION

By the nature of origin, speed stabilization is divided into two components – constructive and tire.

In May 1896, the French inventor Arthur Krebs patented a car with a new way of constructive speed stabilization of steering wheels, which is as follows. Due to the stabilization shoulder, which is created by shifting the pivot forward or its positive longitudinal tilt, lateral reactions R_Y create stabilizing moments relative to the axes of rotation of the steered wheels due to the location of the tire contact patch behind the pivot point of the axis of the pivot shaft. Sometimes the longitudinal inclination of a kingpin is combined with or slightly offset e from the center of the wheel (fig 1a).

To simplify the comparative calculation we take the following assumptions:

1) The steering trapezoid is considered ideal, that is, a geometric slip is absent at all angles of rotation, therefore, the center of rotation always lies on the continuation of the perpendicular to the longitudinal center line of the car, drawn through the center of the rear carriage;

2) The transverse eccentricity of the center of mass is absent, the roll of the sprung masses is not taken into account.

Then the kinematic scheme of rotation (fig 2) is greatly simplified.

To draw the kinematic characteristic, the angle of rotation of the inner steerable wheel α_i is taken as the independent variable.

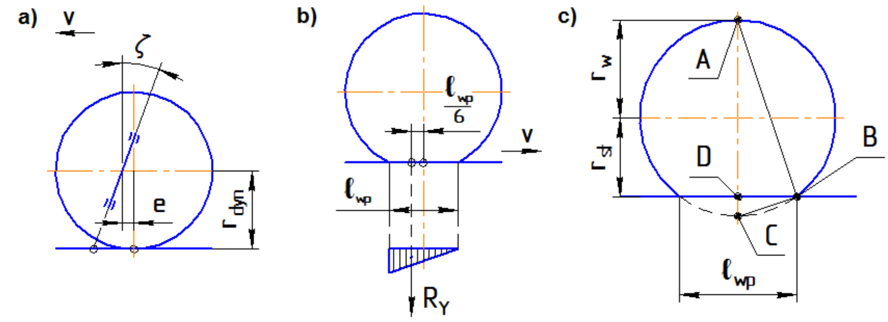

Fig 1. *The moment of speed stabilization. Calculation schemes*

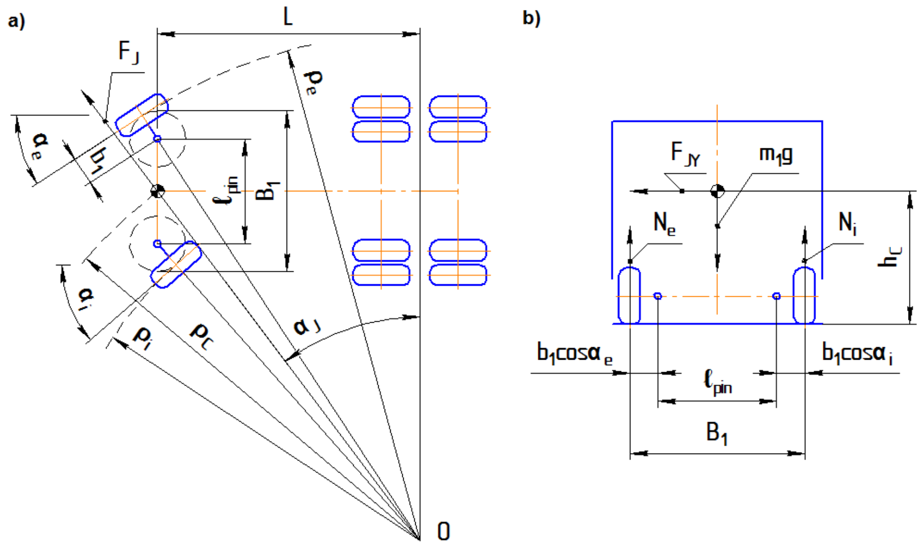

Fig 2. *Kinematic (a) and force (b) rotation patterns*

Distance between the pins:

$$\lambda_{pin} = B_1 - 2b_1 = 2{,}026 - 2 \cdot 0{,}13 = 1{,}766 \text{ m.}$$

The scrub radius r_f is found by the known formula [9]:

$$r_f = b_1 - \frac{\pi \cdot \chi}{180^o} r_{st}, \text{ m}$$

Where $\chi = \beta + \xi$ is substituted in degrees.

Instant turning radius of the inner steering wheel:

$$\rho_i = \frac{L}{\sin \alpha_i} - b_1, \text{ m}$$

Given the first assumption:

$$L = (\rho_i + b_1) \cdot \cos\alpha_i \cdot tg\alpha_i = (\rho_i + b_1) \cdot \sin\alpha_i,$$

$$L = [(\rho_i + b_1) \cdot \cos\alpha_i + \lambda_{pin}] \cdot tg\alpha_e$$

$$L = [(\rho_i + b_1) \cdot \cos\alpha_i + 0{,}5 \cdot \lambda_{pin}] \cdot tg\alpha_J.$$

From here we determine the angle α_e of rotation of the outer steerable wheel and the angle

α_J between the line of action F_J and the normal, lowered from the center of rotation to the longitudinal axis of symmetry of the vehicle, as a function of α_i:

$$\alpha_e = \operatorname{arctg}\frac{(\rho_i + b_1)\cdot \sin\alpha_i}{(\rho_i + b_1)\cdot \cos\alpha_i + \lambda_{pin}};$$

$$\alpha_J = \operatorname{arctg}\frac{(\rho_i + b_1)\cdot \sin\alpha_i}{(\rho_i + b_1)\cdot \cos\alpha_i + 0{,}5\cdot\lambda_{pin}}.$$

Instant radii of rotation of the outer driven wheel and the center of mass of the front axle:

$$\rho_e = \frac{L}{\sin\alpha_e} + b_1,\ \rho_C = \frac{L}{\sin\alpha_J},\ \text{m.}$$

Inertia force attributable to the controlled axis:

$$F_J = \frac{m_1 v^2}{\rho_C},\ \text{N.}$$

Transverse component of inertia force:

$$F_{JY} = F_J \cdot \cos\alpha_J,\ \text{H.}$$

In a curvilinear motion, there is a redistribution of weight on the wheels of the outer side. To determine the ratio of normal N_e reactions on the outer and N_i on the inner steerable wheel, we consider the system of equations of moments relative to the points of contact of the wheels with the supporting surface in the transverse plane (see fig 1). Normal reactions on the outer and inner steering wheels:

$$N_e = \frac{m_1\cdot g\cdot(0{,}5\cdot\lambda_{pin} + b_1\cdot\cos\alpha_e) + F_{JY}\cdot h_C}{\lambda_{pin} + b_1\cdot(\cos\alpha_e + \cos\alpha_i)},$$

$$N_i = \frac{m_1\cdot g\cdot(0{,}5\cdot\lambda_{pin} + b_1\cdot\cos\alpha_i) - F_{JY}\cdot h_C}{\lambda_{pin} + b_1\cdot(\cos\alpha_e + \cos\alpha_i)},\ \text{N.}$$

The side reaction on the wheel is equal to the centrifugal force attributable to it:

$$R_Y = \frac{N}{g}\cdot\frac{v^2}{\rho},\ \text{m,}$$

Where ρ is the instantaneous turning radius of the wheel.

Radial tire stiffness:

$$C_{TZ} = \frac{m_{Tnom}\cdot g}{r_w - r_{st}} = \frac{2240\cdot 9{,}81}{0{,}509 - 0{,}475} = \frac{21974{,}4}{0{,}509 - 0{,}475} = 646305{,}88\ \text{N/m.}$$

For a solid support surface, we can take $r_{dyn}=r_{st}$ and write the dynamic radius as a function of the actual normal load on the wheel N, taking into account the redistribution of weight:

$$r_{dyn} = f(N) = r_{st} = r_w - \frac{N}{C_{TZ}},\ \text{m.}$$

The mechanism of the tire speed stabilization is as follows. From the theory of lateral slip it is known that when rotated under the action of lateral forces, an elastic deformation of the tire occurs, accompanied by the redistribution of the specific pressure and tangential stresses. The contact patch, which has the shape of an ellipse in rectilinear motion, becomes bean-shaped. Elementary side reactions are distributed so that the line of action of the resulting side reaction R_Y does not coincide with the center of the contact patch, but is shifted back by a distance approximately equal to $\ell_{wp}/6$ (fig 1b), where ℓ_{wp} is length of the contact patch, defined by the following formula (fig 1c):

$$\lambda_{wp} = 2\sqrt{r_w^2 - r_{st}^2}\text{ , m.}$$

In general, the moment of speed stabilization is as follows:

$$M_{Sv} = R_Y \cdot \left(r_{dyn} \cdot \sin\zeta + e + \frac{\lambda_{wp}}{6} \right)\text{, Nm.}$$

The lateral response of R_Y increases with increasing speed v and increasing slip angle. Reducing the coefficient of adhesion slows growth and reduces the amplitude of R_Y. With full lateral sliding, the plot of R_Y takes the form of a rectangle, and ℓ_{pw}=0. When calculating the moment from the speed stabilization, it is necessary to take into account that this technique is applicable only in the subcritical velocity range.

Critical speed by tipping:

$$v_{cr.h} = \sqrt{\frac{\rho_C \cdot g}{h_C} \cdot \frac{B_1}{2}}\text{ , m/s}^2\text{.}$$

Critical speed by slip:

$$v_{cr.\mu} = \sqrt{\rho_i \cdot g \cdot \mu_Y}\text{ , m/s}^2\text{,}$$

Where μ_Y is the coefficient of adhesion in the transverse direction.

The dependences of the critical speeds on the angle of rotation of the inner steerable wheel for the initial data are shown in fig 3, the dependences of the moment of speed stabilization for different speeds are shown in fig 4.

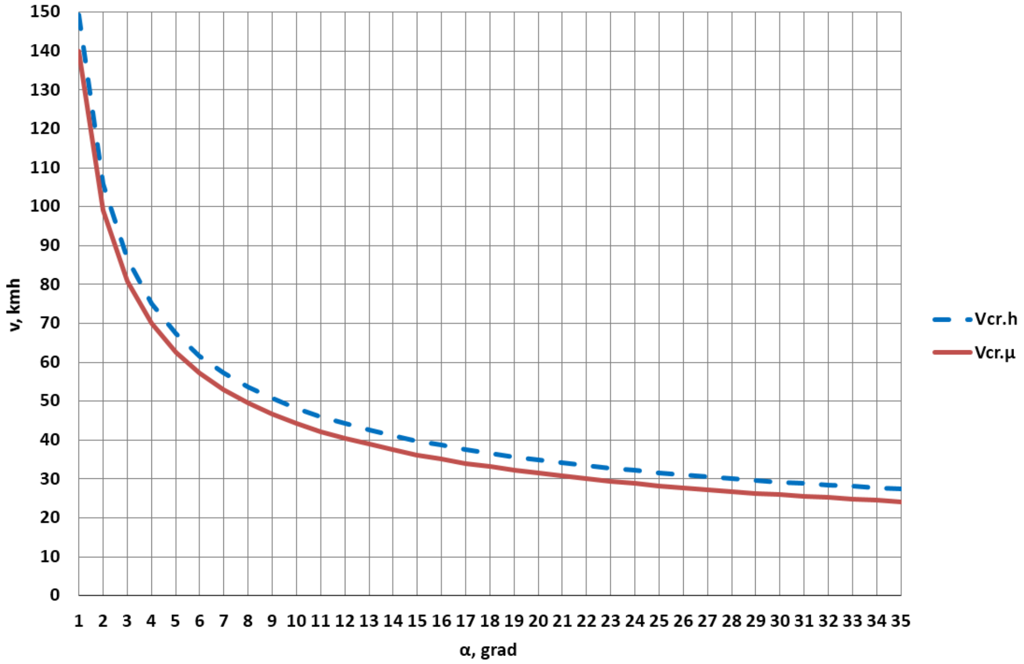

Fig 3. *Dependence of critical speed on wheels rotation angle*

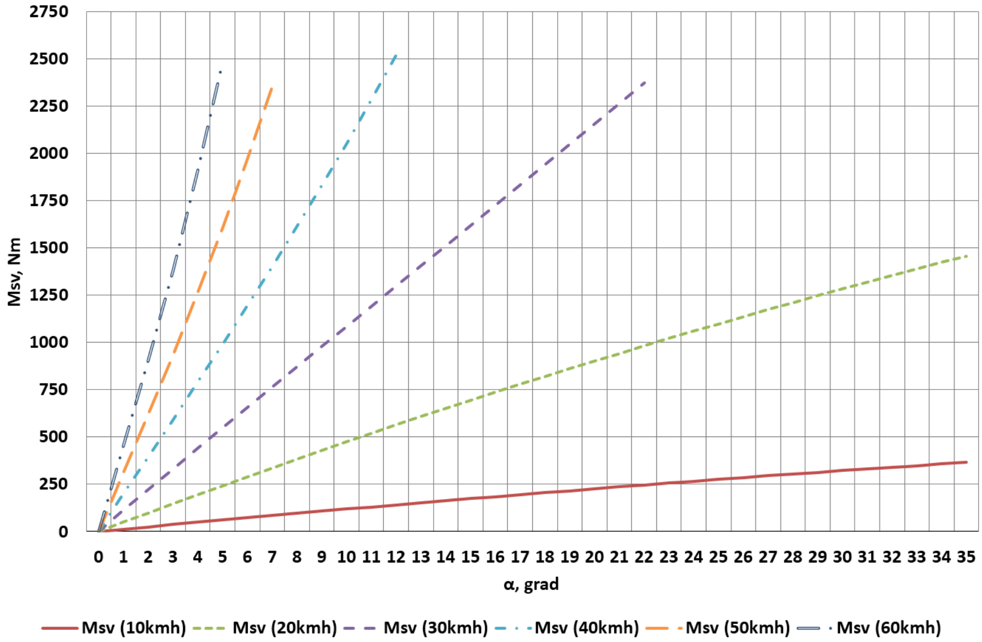

Fig 4. *Dependence of speed stabilization moment on wheels rotation angle*

IV. WEIGHT STABILIZATION

The mechanism of weight stabilization is as follows. When turning relative to the inclined axis, the trunnion describes an arc in a plane deviated from the horizontal position, as a result of which the wheel tends to descend below the plane of the road, and since this is impossible, the front end of the body rises. When exiting the turn, the steered wheels tend to return to the neutral position under the weight of the car. The potential energy of the body is equal to the work of the stabilizing moment M_{Sg} and increases in proportion to the height change Δh:

In the literature, you can find two formulas for calculating Δh:

$$\Delta h_1 = 0{,}5 \cdot r_f \cdot \sin\xi \cdot \sin\alpha - r_{st} \cdot \sin^2\zeta \cdot \sin\alpha \text{, m [9];}$$

$$\Delta h_2 = r_f\left(\xi \cdot \sin\alpha + \zeta \cdot \cos\alpha\right) \text{, m [10].}$$

Traditionally, the lifting height is decomposed into two components – from the kingpin lateral inclination (h_ξ) and from the camber (h_β) [7]:

$$\Delta h_3 = r_f \cdot \sin\xi \cdot \cos\xi \cdot (1-\cos\alpha) + \sin\beta \cdot b_1 \cdot \sin\alpha \text{, m.}$$

Despite the fact that when calculating according to this formula, the graph is more realistic, this method of calculation does not take into account the above-mentioned **influence of ζ on the weight stabilization**. The refined formula is as follows:

$$\Delta h_4 = r_f \cdot \sin\chi \cdot \cos\chi \cdot (1-\cos\alpha) + \sin\zeta \cdot b_1 \cdot \sin\alpha \text{, m,}$$

Where for the outer wheel substitute $-\alpha_e$.

However, the Δh_4 value is **underestimated**. Therefore, the author has developed a new method for calculating the moment of weight stabilization, based on the canonical equation of an ellipse [11]. The projection of the pivot pin on the plane of the ellipse is the radius of the base of the cone, which it describes (fig 5):

$$b_{1Y} = b_1 \cdot \cos\chi \cdot$$

The Y coordinate of the C point: $y = b_{1Y} \cdot \sin\zeta \cdot \sin\alpha$.

Taking into account the rules of signs:

$$y_i = b_1 \cdot \cos\chi \cdot \sin\zeta \cdot \sin\alpha_i;$$

$$y_e = b_1 \cdot \cos\chi \cdot \sin\zeta \cdot \sin(-\alpha_e).$$

Let us consider an ellipse, which is a projection of the base of a cone.

The inverse of the compression ratio of the ellipse:

$$\frac{r_2}{r_1} = \frac{b_1 \cdot \cos\zeta \cdot \cos\chi}{b_1 \cdot \sin\zeta} = \frac{\cos\chi}{tg\zeta}.$$

The canonical equation of an ellipse is:

$$\frac{x^2}{r_2^2} + \frac{y^2}{r_1^2} = 1 \cdot$$

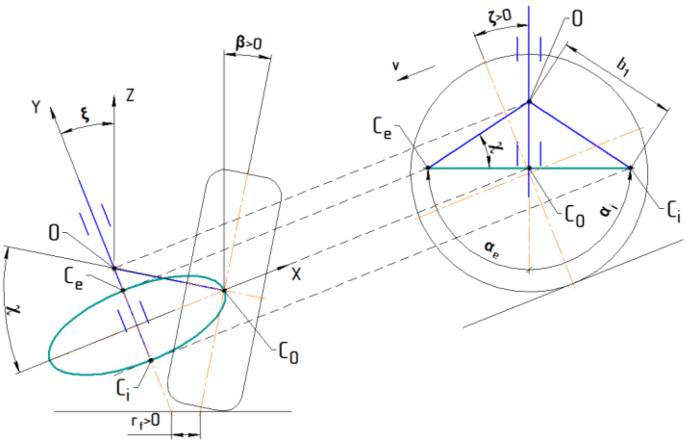

Fig 5. *Influence of the kingpin lateral inclination on the trunnion trajectory*

From here we get the coordinate of point C along the X axis:

$$x = \cos\chi\sqrt{(b_1 \cdot \cos\zeta)^2 - (y \cdot ctg\zeta)^2}, \text{ m.}$$

From analytical geometry it is known that when the coordinate system rotates by an angle ξ without changing its origin, the coordinates are transformed as follows:

$$\begin{cases} x_1 = x \cdot \cos\xi + y \cdot \sin\xi \\ y_1 = -x \cdot \sin\xi + y \cdot \cos\xi \end{cases},$$

Where x, y - the initial coordinates of the point; x_1, y_1 - coordinates of a point in a rotated coordinate system.

Accordingly, the coordinate point C along the Z axis: $z = -x \cdot \sin\xi + y \cdot \cos\xi$, m.

Lifting the front of the car: $\Delta h_5 = z - z_0$, m,

Where z_0 is the height of the body when α = 0.

Figure 6 shows the graphs of dependences $\Delta h_{1...5}$ on the angle of rotation of the steerable wheel α. It is evident that formulas for $\Delta h_{1...2}$ do not sustain any critics from the point of view of the physics of the process.

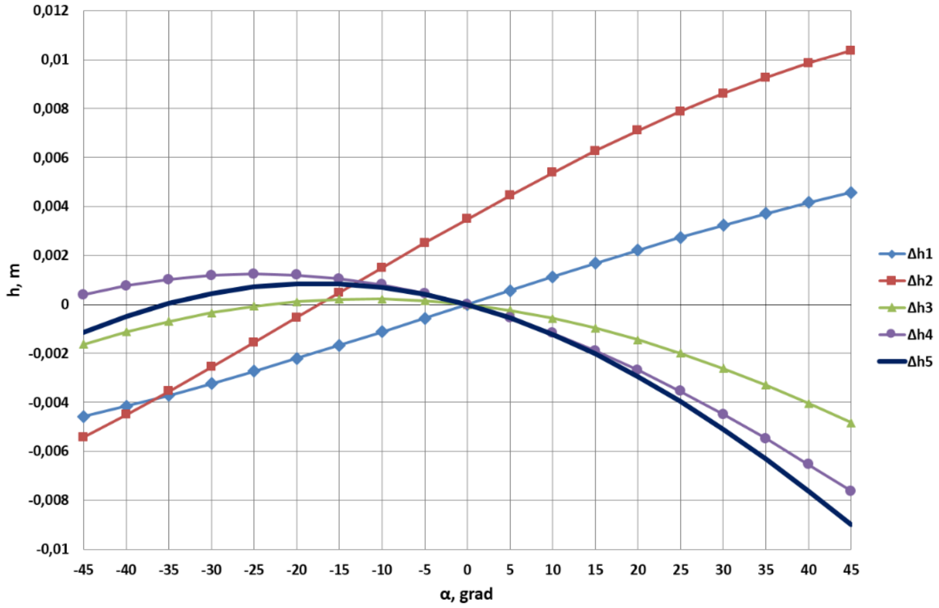

Fig 6. *Dependence of the body lift height on wheel rotation angle*

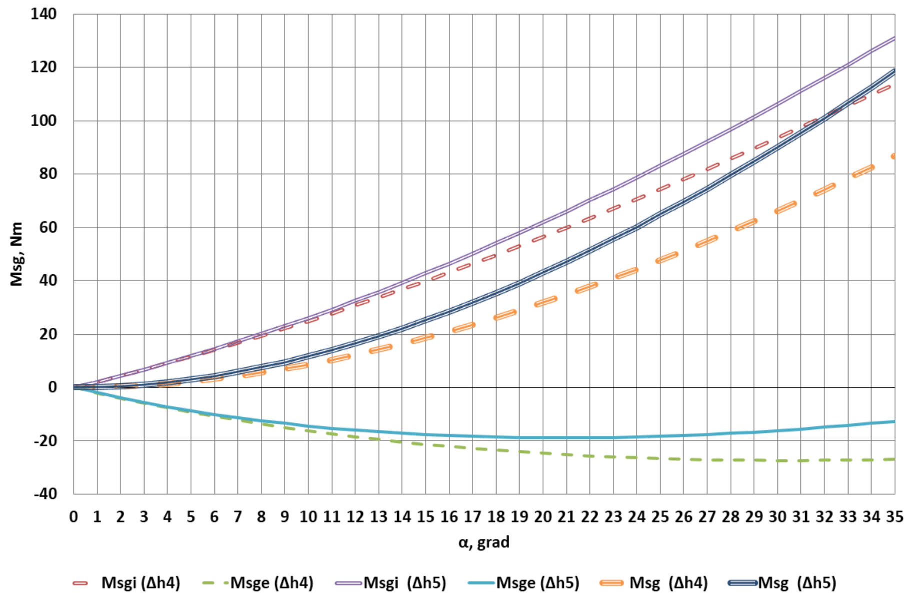

Fig 7. *Dependence of weight stabilization moment on wheels rotation angles*

Total stabilizing moment: $M_{S\Sigma} = M_{Sg} + M_{SV}$.

The percentage of the components of the total stabilizing moment at $\alpha_i = 35°$ for different speeds is presented in table 1, considering that for a given angle of rotation the critical speed by slip $v_{cr.\mu} = 24$ kmh.

Table 1. *The ratio of the components of the stabilizing moment*

v, kmh	$M_{S\Sigma}$, Nm	M_{Sg}, Nm	M_{Sv}, Nm	M_{Sg}:M_{Sv}, %
0	122,76	122,76	0	100:0
5	210,88	118,46	92,42	56:44
10	471,31	105,56	365,75	22:78
15	899,21	84,06	815,15	9:91
20	1510,93	53,97	1456,96	4:96

V. CONCLUSION

Analysis of calculations based on the developed mathematical model allows us to draw the following conclusions:

1. Taking into account the redistribution of weight when calculating the moment of speed stabilization has little effect on the result at v up to 20 kmh (the error is no more than 1%). With increasing speed, the error of the previous technique increases significantly, reaching 11...14% in the range v = 40...60 kmh.

2. $\Delta h_e < 0$ (see fig 6) means that the forward axle roll in the direction coincides with the body roll. This phenomenon adversely affects the stability of the movement.

3. M_{Sge} at $\Delta h_e < 0$ becomes negative, i.e. destabilizing (see fig 7). Turned wheel tends to turn to an even greater angle. As a result, the total M_{Sg} will decrease.

4. With increasing speed, the total moment of the weight stabilization M_{Sg} decreases, since the weight is redistributed to the outer wheel, rotated at a smaller angle (see table 1).

5. When v > 5 kmh, speed stabilization begins to prevail over weight stabilization, and the higher the speed, the more significant (see table 1).

To date, there are no mathematical models that allow us to uniquely calculate the values of the installation angles of the steerable wheels, since they strongly depend on the driving mode and the normal load on the wheels. In this case, the correction of one parameter simultaneously causes a change in other parameters, each of which affects several vehicle characteristics. The improvement of one of those is often accompanied by the deterioration of the other. Thus, the choice of the installation angles of the steerable wheels is a complex task, aimed at finding the optimum and solved by an iterative method [12]. The solution begins with a kinematic calculation for different driving conditions. Having achieved acceptable results of theoretical calculations, they produce an experimental refinement of the car, performing a large number of special tests, evaluating the objective parameters recorded by the sensors, and the subjective feelings of the test pilots.

The mathematical model developed by the author, taking into account the influence of the Caster angle on the weight stabilization, and the redistribution of weight during curvilinear motion, will allow to refine the existing design algorithms for calculating the design characteristics of the suspension and steering and reduce the amount of experimental developmental tests.

References

[1]. *Kristal'nyj S.R., Popov N.V.* Avtotransportnoe predprijatie, 2007, No. 9, pp. 46-49.

[2]. *Demidov L.V.* Vestnik MADI, 2013, Issue 4, pp. 17a-21.

[3]. *Kristal'nyj S.R., Toporkov M.A., Fomichjov V.A., Popov N.V. Avtomobil'*. Doroga. Infrastruktura, 2015, No. 2, p. 2.

[4]. *Kotovich S.V.* Vestnik MADI, 2004, Issue 3, pp. 27-33.

[5]. *Malinovsky M.P*. Jeksperimental'noe issledovanie harakteristik sistem upravlenija transportnyh sredstv [Experimental study of control systems characteristics of vehicles]. Moscow, MADI, 2011, 123 p.

[6]. *Ivanov A.M., Losev S.A.* Vestnik MADI, 2005, Issue 5, pp. 32-37.

[7]. *Malinovsky M.P*. Sistemy upravleniya kolesnyh mashin [Control systems of wheeled machines]. Moscow, MADI, 2018, 100 p.

[8]. *Gladov G.I. et al.* Konstrukcii mnogocelevyh gusenichnyh i kolesnyh mashin [Designs for multi-purpose caterpillar and wheeled vehicles]. Moscow, Akademija, 2010, 400 p.

[9]. *Grishkevich A.I. et al*. Avtomobili. Konstrukciya, konstruirovanie i raschet. Sistemy upravleniya i hodovaya chast' [Cars. Design, construction and calculation. Control systems and chassis]. Minsk: Vyshehjshaya shkola, 1987, 199 p.

[10]. *Afanas'ev B.A., Belousov B.N., Zheglov L.F. et al*. Proektirovanie polnoprivodnyh kolesnyh mashin [Design of all-wheel drive wheeled vehicles], Vol. 3. Moscow, Izd-vo MGTU im. N.E. Baumana, 2008, 432 p.

[11]. *Malinovsky M.P*. Vestnik MADI, 2018, Issue 3, pp. 21-29.

[12]. *Malinovsky M.P*. Traktory i sel'hozmashiny, 2015, No. 8, pp. 17–19.